$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

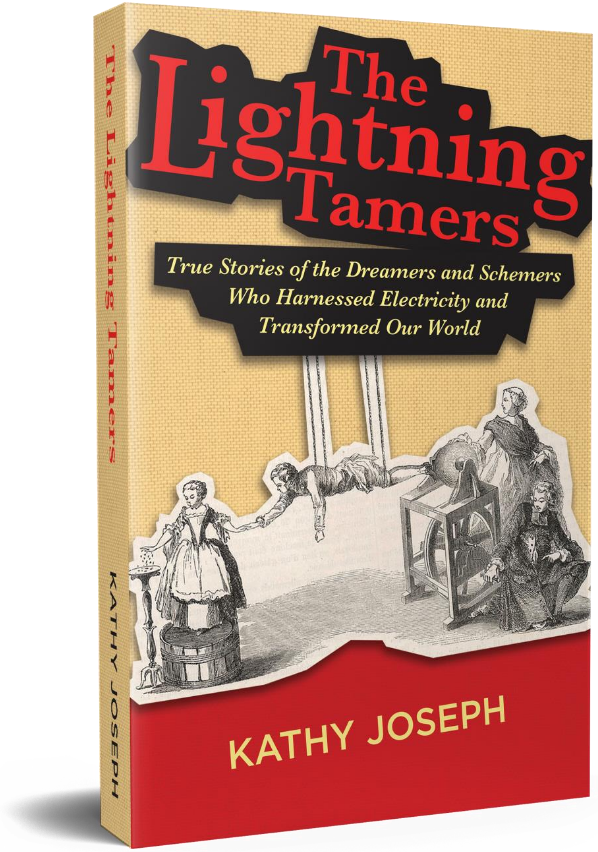
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

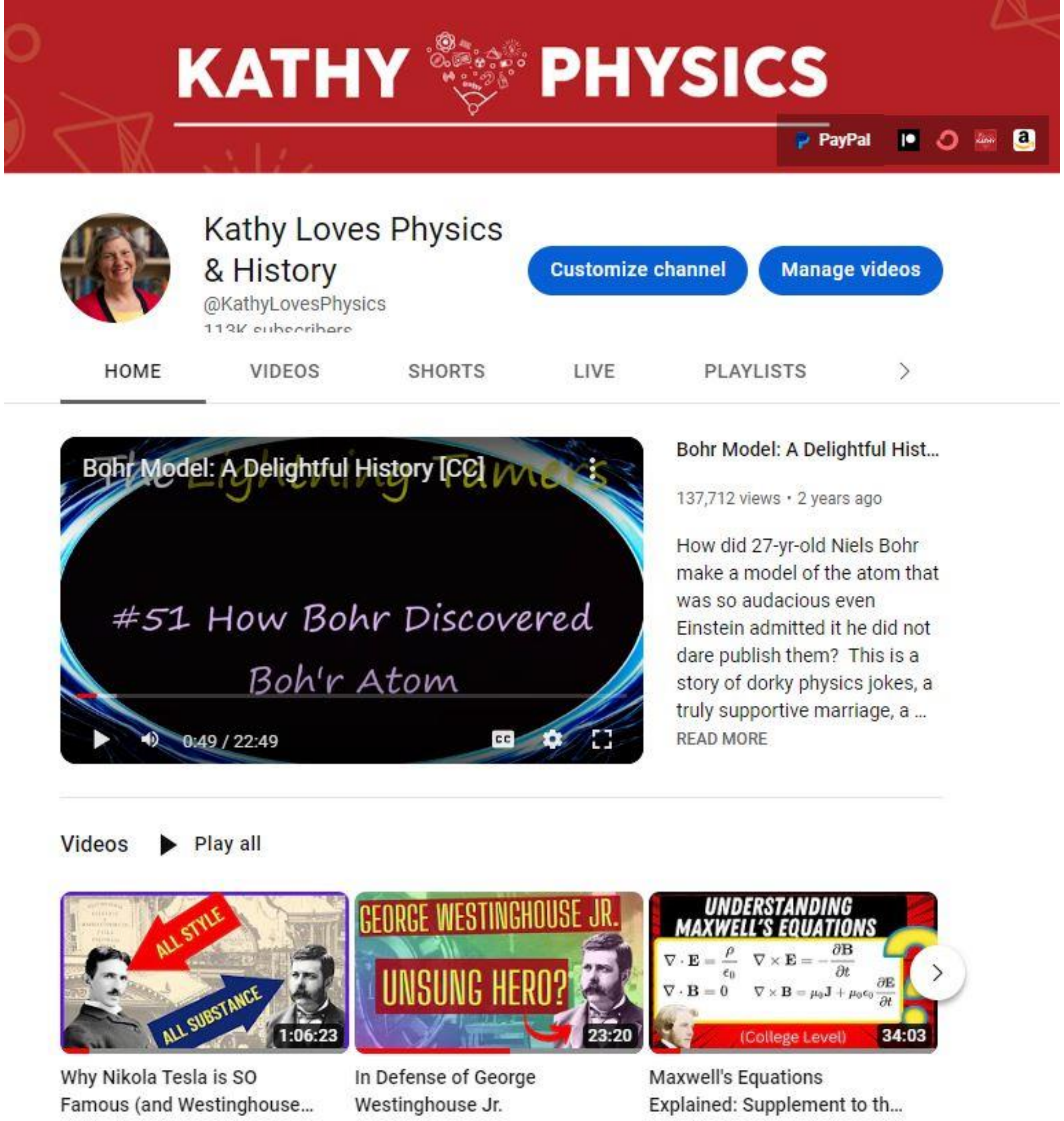
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

# A Whirlwind History of Maxwell’s Equations

Kathy Joseph

# A bit about me:



Tesla Coil at the Spark Museum

Me in "Cage of Doom"

# ELEVENTH SERIES.

§ 18. *On Induction.* ¶ i. *Induction an action of contiguous particles.* ¶ ii. *Absolute charge of matter.* ¶ iii. *Electrometer and inductive apparatus employed.* ¶ iv. *Induction in curved lines.* ¶ v. *Specific inductive capacity.* ¶ vi. *General results as to induction.*

Received November 30,—Read December 21, 1837.

¶ i. *Induction an action of contiguous particles.*

1161. The science of electricity is in that state in which every part of it requires experimental investigation ; not merely f
the discovery of new effects, but what is just now of far m
importance, the development of the means by which th
effects are produced, and the consequent more accurate
mination of the first principles of action of the most extra
nary and universal power in nature :—and to those philoso
who pursue the inquiry zealously yet cautiously, combini

# Outline

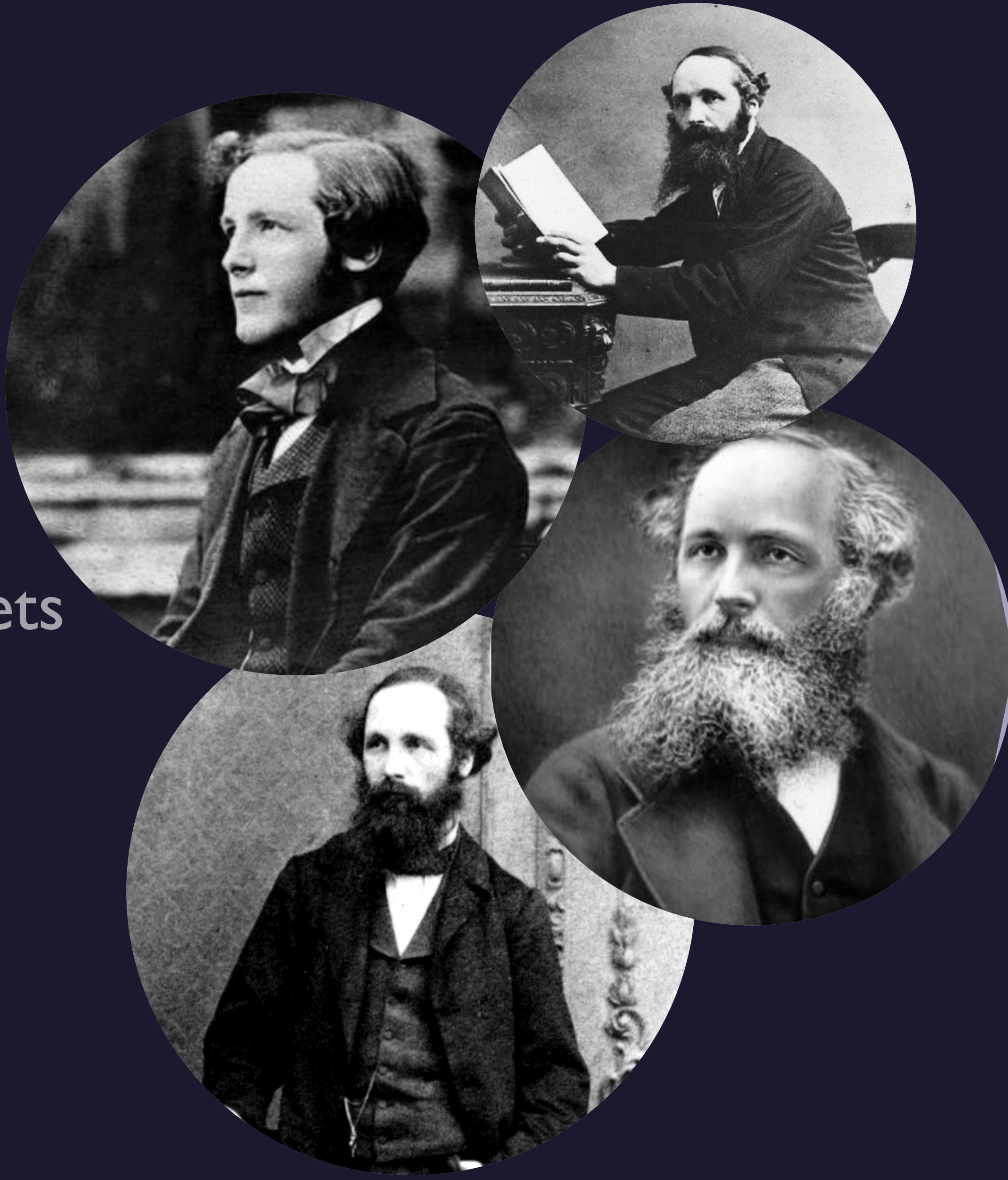
Part 1:  Gauss's Law

Part 2:  Ampere's Law w/ Maxwell's Addition

Part 3: Faraday's Law + Gauss's Law for Magnets

Part 4: Faraday, Maxwell + Light

Part 5: Quaternions & Why 1864



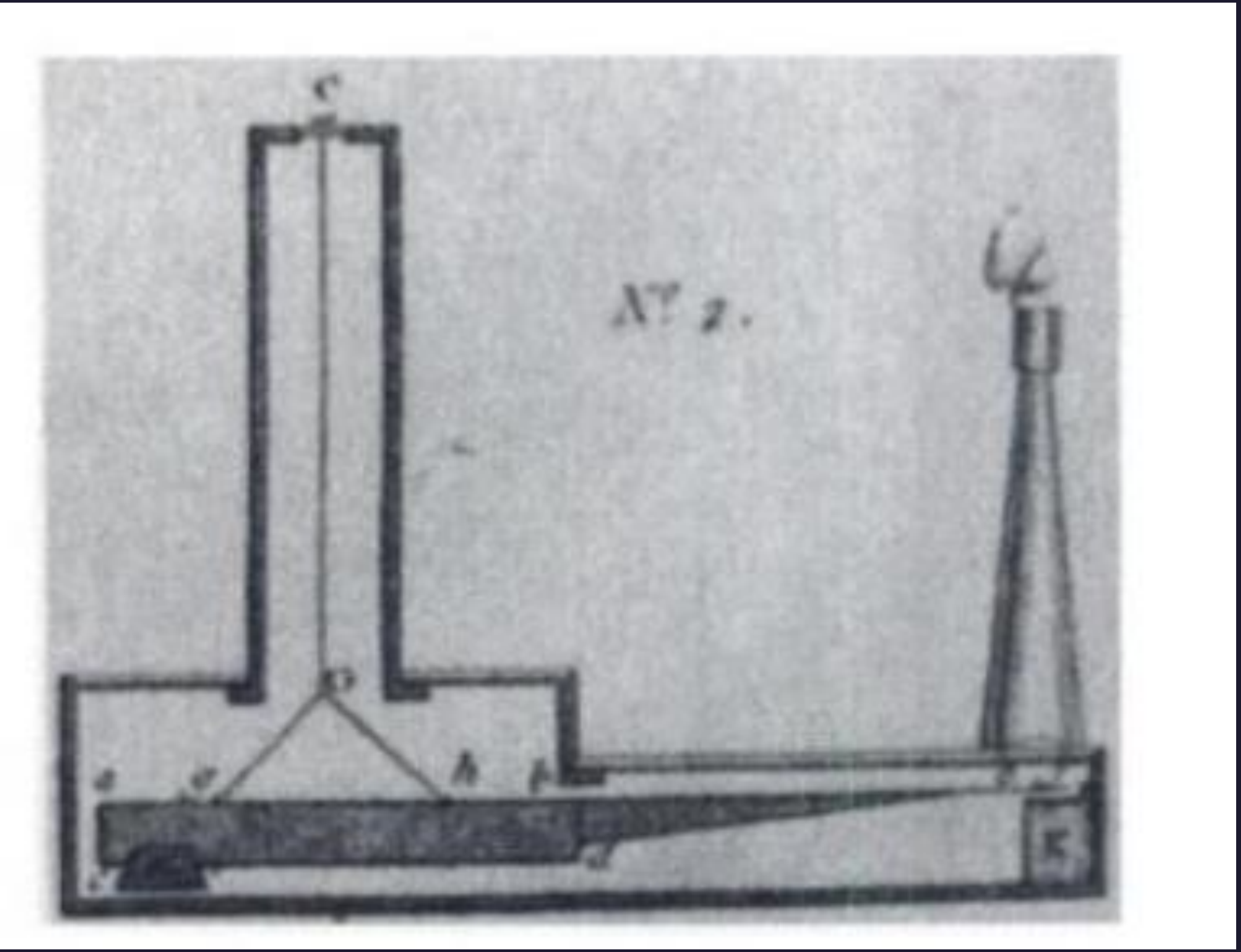
# Part 1

## Gauss's Law

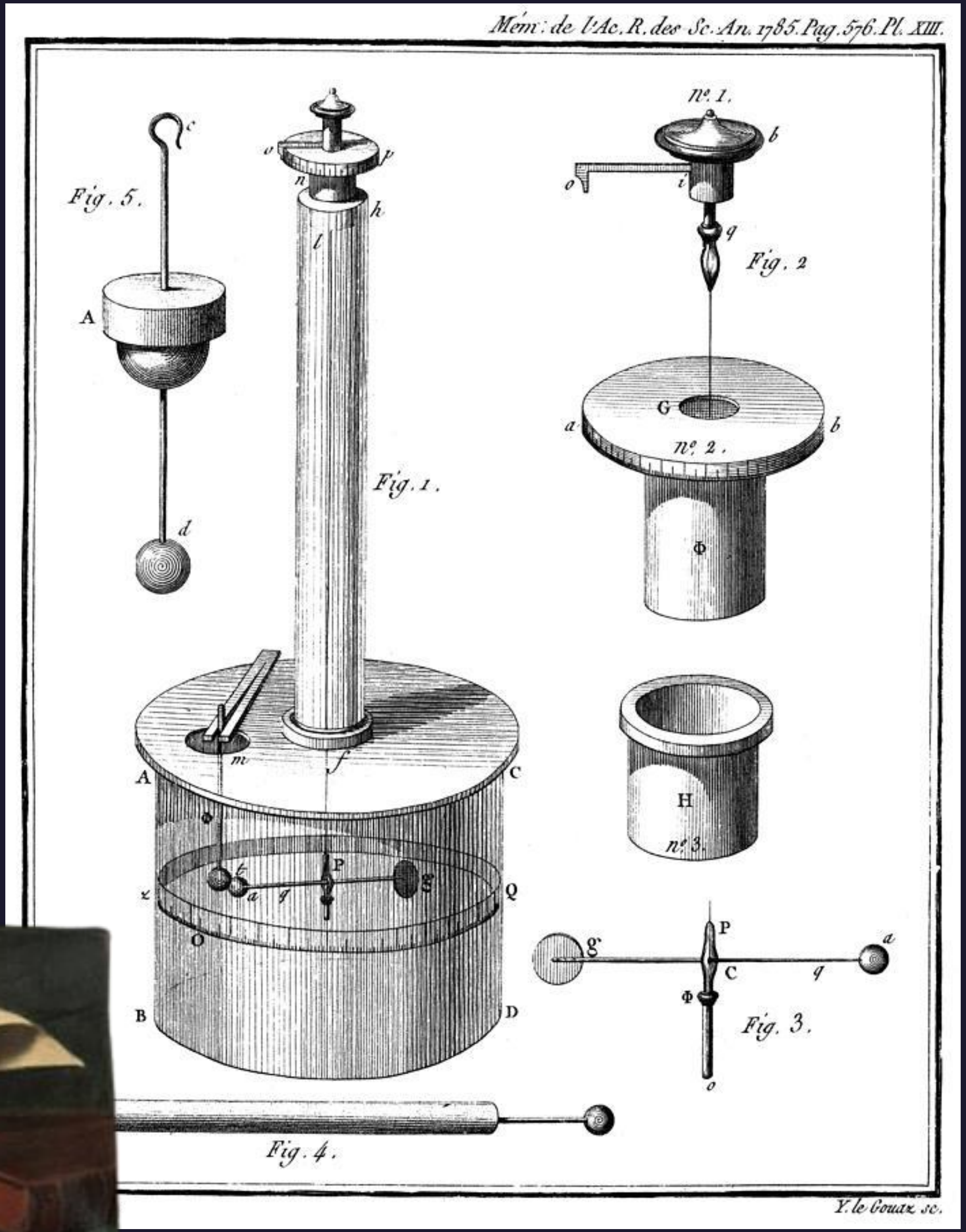
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

**Figure 9.2** Schematic view of Coulomb's magnetic compass. (Photo courtesy of the Burndy Library, Dibner Institute for the History of Science and Technology.)

# Early 1780s



# Torsional Balance
# 1784



# 1785



## PREMIER MÉMOIRE SUR L'ÉLECTRICITÉ ET LE MAGNÉTISME.

Par M. COULOMB.

*Construction & usage d'une Balance électrique, fondée sur la propriété qu'ont les Fils de métal, d'avoir une force de réaction de Torsion proportionnelle à l'angle de Torsion.*

*Détermination expérimentale de la loi suivant laquelle les élémens des Corps électrisés du même genre d'Électricité, se repoussent mutuellement.*

DANS un Mémoire donné à l'Académie, en 1784, j'ai déterminé, d'après l'expérience, les loix de la …

… e dans la formule qui exprime le frottement de la … ce d'un corps solide en mouvement dans un fluide. … mets aujourd'hui sous les yeux de l'Académie, une … ce électrique construite d'après les mêmes principes;





du fil de suspension & la balle *t*, l'on rencontre la première division *o* du cercle *zoq*. La balance est actuellement en état de se prêter à toutes les opérations; nous allons en donner pour exemple, le moyen dont nous nous sommes servi pour déterminer la loi fondamentale suivant laquelle les corps électrisés se repoussent.

### *Loi fondamentale de l'Électricité.*

*La force répulsive de deux petits globes électrisés de la même nature d'électricité, est en raison inverse du carré de la distance du centre des deux globes.*

### EXPÉRIENCE.

L'on électrise, *fig. 4*, un petit conducteur, qui n'est autre chose, qu'une épingle à grosse tête, qui se trouve isolée en enfonçant sa pointe dans l'extrémité d'un bâton de cire d'Espagne; l'on introduit cette épingle dans le trou *m*, & on lui fait toucher la balle *t*, en contact avec la balle *a*: en retirant l'épingle, les deux balles se trouvent électrisées de la même nature d'électricité, & elles se …

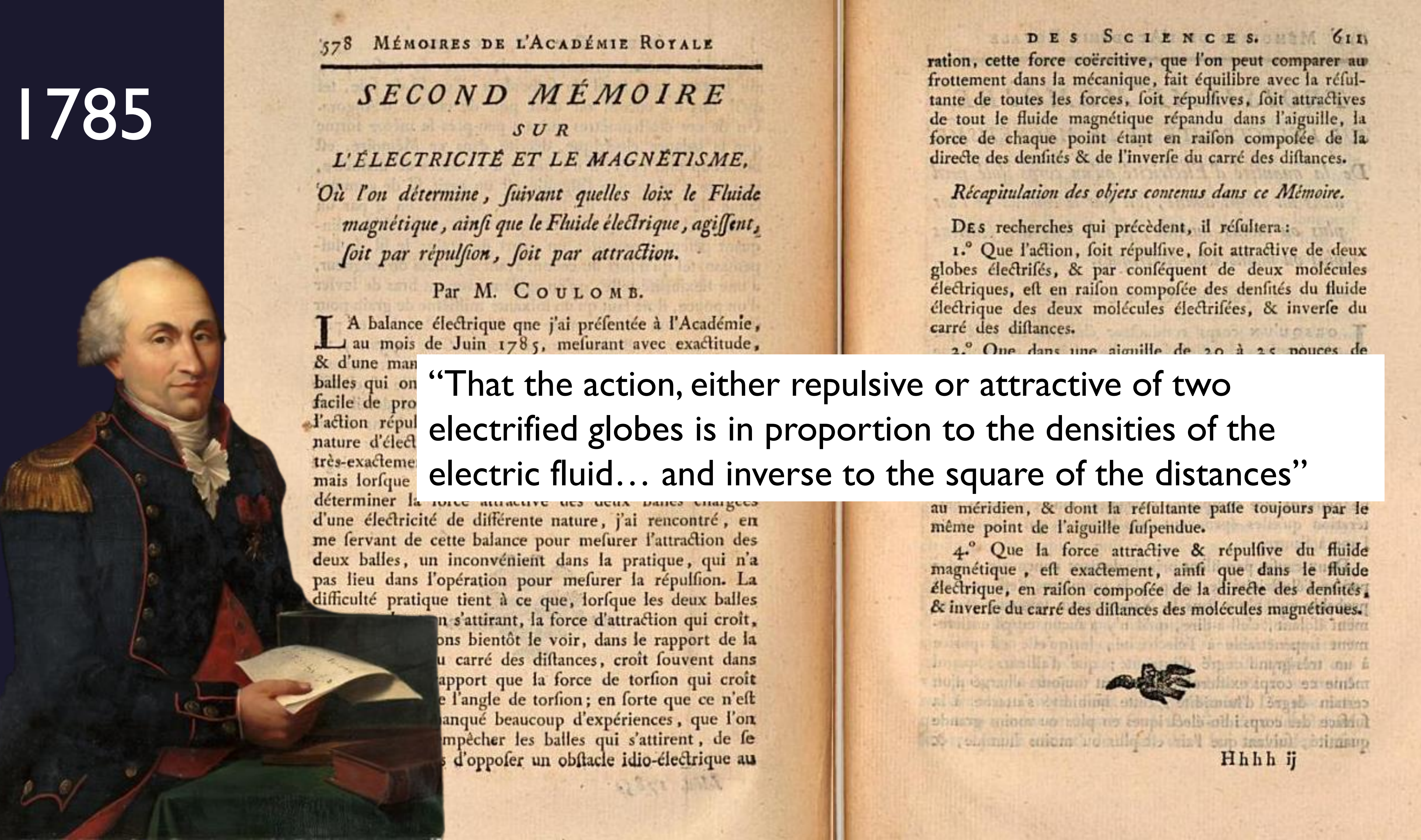
… Je présenterai seulement ici, quelques essais qui sont faciles à répéter, & qui mettront tout de suite sous les yeux, la loi de la répulsion.

*Premier Essai.* Ayant électrisé les deux balles avec la

*The fundamental law of Electricity.*

*The repulsive force of two small globes electrified with the same kind of electricity, is inversely proportional to the square of the distance between the centers of the two globes.*

# 1785



578 MÉMOIRES DE L'ACADÉMIE ROYALE

SECOND MÉMOIRE SUR L'ÉLECTRICITÉ ET LE MAGNÉTISME,

*Où l'on détermine, suivant quelles loix le Fluide magnétique, ainsi que le Fluide électrique, agissent, soit par répulsion, soit par attraction.*

Par M. COULOMB.

LA balance électrique que j'ai présentée à l'Académie, au mois de Juin 1785, mesurant avec exactitude, & d'une man… balles qui on… facile de pro… l'action répul… nature d'élect… très-exactemen… mais lorsque … déterminer la force attractive des deux balles chargées d'une électricité de différente nature, j'ai rencontré, en me servant de cette balance pour mesurer l'attraction des deux balles, un inconvénient dans la pratique, qui n'a pas lieu dans l'opération pour mesurer la répulsion. La difficulté pratique tient à ce que, lorsque les deux balles …n s'attirant, la force d'attraction qui croît, …ons bientôt le voir, dans le rapport de la …u carré des distances, croît souvent dans …apport que la force de torsion qui croît …e l'angle de torsion; en sorte que ce n'est …anqué beaucoup d'expériences, que l'on …mpêcher les balles qui s'attirent, de se …s d'opposer un obstacle idio-électrique au

DES SCIENCES. 611

ration, cette force coërcitive, que l'on peut comparer au frottement dans la mécanique, fait équilibre avec la résultante de toutes les forces, soit répulsives, soit attractives de tout le fluide magnétique répandu dans l'aiguille, la force de chaque point étant en raison composée de la directe des densités & de l'inverse du carré des distances.

*Récapitulation des objets contenus dans ce Mémoire.*

DES recherches qui précèdent, il résultera:

1.° Que l'action, soit répulsive, soit attractive de deux globes électrisés, & par conséquent de deux molécules électriques, est en raison composée des densités du fluide électrique des deux molécules électrisées, & inverse du carré des distances.

2.° Que dans une aiguille de 20 à 25 pouces de …

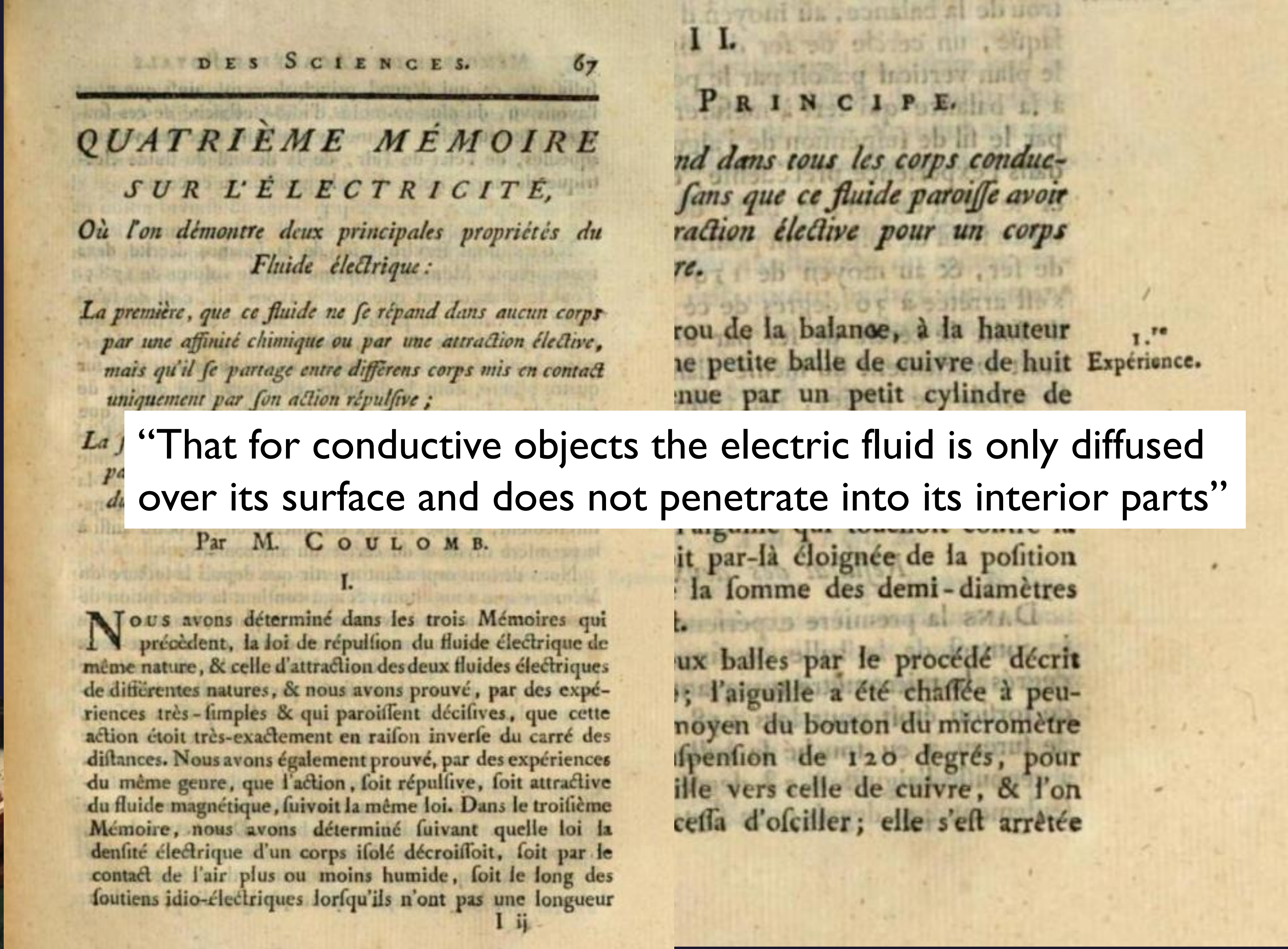
… au méridien, & dont la résultante passe toujours par le même point de l'aiguille suspendue.

4.° Que la force attractive & répulsive du fluide magnétique, est exactement, ainsi que dans le fluide électrique, en raison composée de la directe des densités, & inverse du carré des distances des molécules magnétiques.

Hhhh ij

"That the action, either repulsive or attractive of two electrified globes is in proportion to the densities of the electric fluid… and inverse to the square of the distances"

# 1786





QUATRIÈME MÉMOIRE
SUR L'ÉLECTRICITÉ,

*Où l'on démontre deux principales propriétés du Fluide électrique :*

*La première, que ce fluide ne se répand dans aucun corps par une affinité chimique ou par une attraction élective, mais qu'il se partage entre différens corps mis en contact uniquement par son action répulsive ;*

Par M. COULOMB.

I.

NOUS avons déterminé dans les trois Mémoires qui précèdent, la loi de répulsion du fluide électrique de même nature, & celle d'attraction des deux fluides électriques de différentes natures, & nous avons prouvé, par des expériences très-simples & qui paroissent décisives, que cette action étoit très-exactement en raison inverse du carré des distances. Nous avons également prouvé, par des expériences du même genre, que l'action, soit répulsive, soit attractive du fluide magnétique, suivoit la même loi. Dans le troisième Mémoire, nous avons déterminé suivant quelle loi la densité électrique d'un corps isolé décroissoit, soit par le contact de l'air plus ou moins humide, soit le long des soutiens idio-électriques lorsqu'ils n'ont pas une longueur



I I.

PRINCIPE.

*nd dans tous les corps conduc-*
*sans que ce fluide paroisse avoir*
*raction élective pour un corps*
*re.*

rou de la balance, à la hauteur 1.re Expérience.
ne petite balle de cuivre de huit
nue par un petit cylindre de

it par-là éloignée de la position
la somme des demi-diamètres
t.

ux balles par le procédé décrit
; l'aiguille a été chassée à peu-
noyen du bouton du micromètre
spension de 120 degrés, pour
ille vers celle de cuivre, & l'on
cessa d'osciller; elle s'est arrêtée

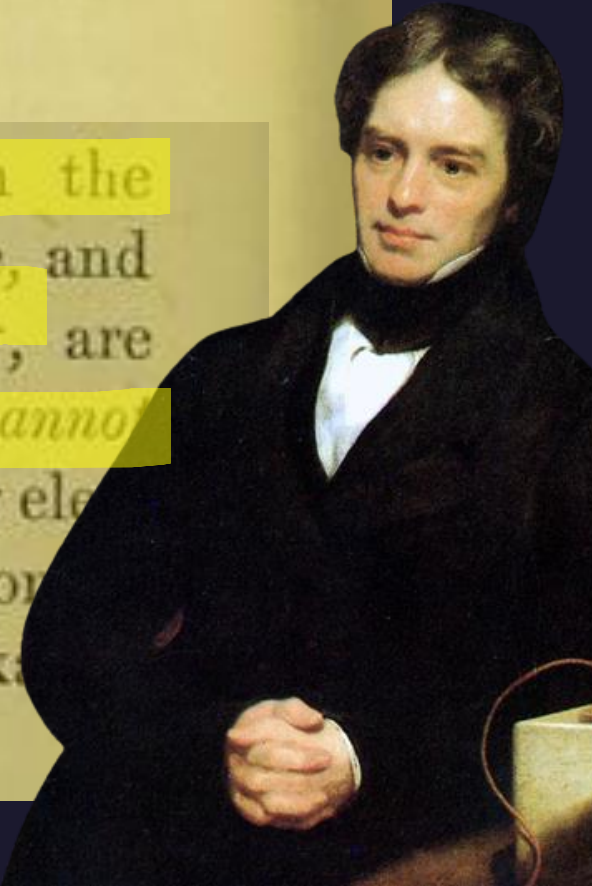
"That for conductive objects the electric fluid is only diffused over its surface and does not penetrate into its interior parts"

## ELEVENTH SERIES.

§ 18. *On Induction.* ¶ i. *Induction an action of contiguous particles.* ¶ ii. *Absolute charge of matter.* ¶ iii. *Electrometer and inductive apparatus employed.* ¶ iv. *Induction in curved lines.* ¶ v. *Specific inductive capacity.* ¶ vi. *General results as to induction.*

Received November 30,—Read December 21, 1837.

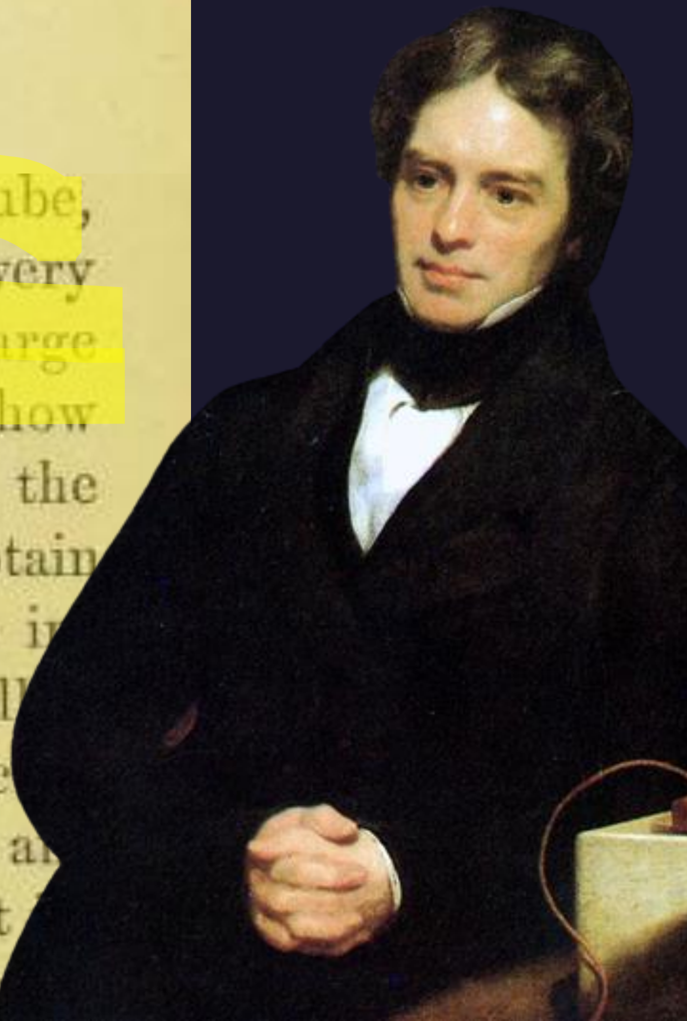
1170. The beautiful experiments of Coulomb upon the equality of action of *conductors*, whatever their substance, and the residence of *all* the electricity upon their surfaces †, are sufficient, if properly viewed, to prove that *conductors cannot be bodily charged*; and as yet no means of communicating ele tricity to a conductor so as to place its particles in relatio one electricity, and not at the same time to the other in ex equal amount, has been discovered.



Received November 30,—Read December 21, 1837.

1173. I carried these experiments on with air to a very great extent. I had a chamber built, being a cube of twelve feet. A slight cubical wooden frame was constructed, and copper wire passed along and across it in various directions, so as to make the sides a large net-work, and then all was covered in with paper, placed in close connexion with the wires, and supplied in every direction with bands of tin foil, that the whole

1174. I put a delicate gold-leaf electrometer within the cube, and then charged the whole by an *outside* communication, very strongly, for some time together; but neither during the charge or after the discharge did the electrometer or air within show the least signs of electricity. I charged and discharged the whole arrangement in various ways, but in no case could I obtain the least indication of an absolute charge; or of one by i
duction in which the electricity of one kind had the small
superiority in quantity over the other. I went into the c
and lived in it, and using lighted candles, electrometers, a
all other tests of electrical states, I could not find the least
fluence upon them, or indication of anything particular give



Received November 30,—Read December 21, 1837

1231. As argument against the received theory of induction and in favour of that which I have ventured to put forth, I cannot see how the preceding results can be avoided. The effects are clearly inductive effects produced by electricity, not in currents but in its statical state, and this induction is exerted in lines of force which, though in many experiments they may be straight, are here curved more or less according to circumstances. I use the term *line of inductive force* merely as a temporary conventional mode of expressing the direction of the power in cases of induction; and in the experiments with the emisphere (1224.), it is curious to see how, when certain lines

Received November 30,—Read December 21, 1837.

others, which makes me desirous of placing the remarks on absolute charge first, in the order of proof and argument, which I am about to adduce in favour of my view, that electric induction is an action of the contiguous particles of the insulating medium or *dielectric**.

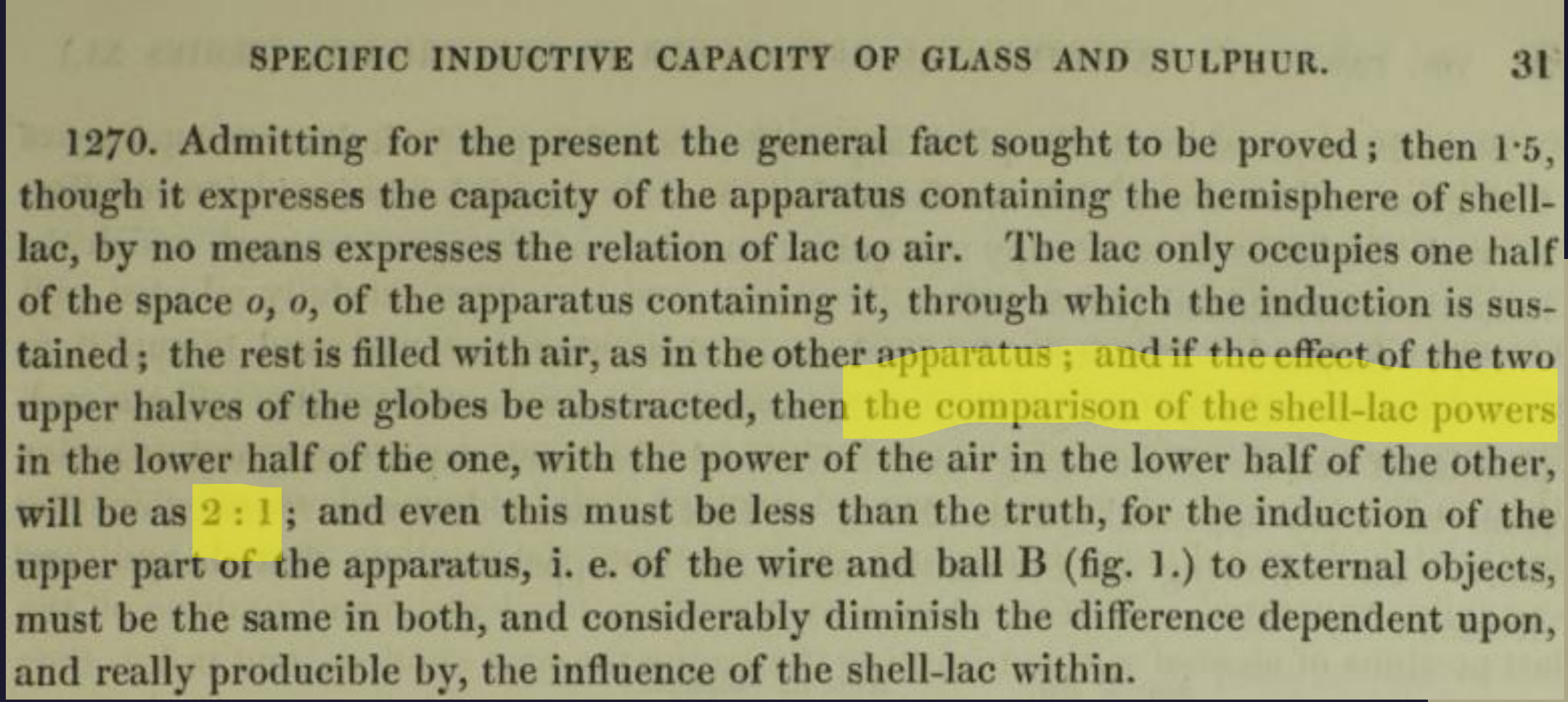
* I use the word *dielectric* to express that substance through or across which the electric forces are acting.—*Dec.* 1838.

Received November 30,—Read December 21, 1837.

¶ v. *On specific induction, or specific inductive capacity.*

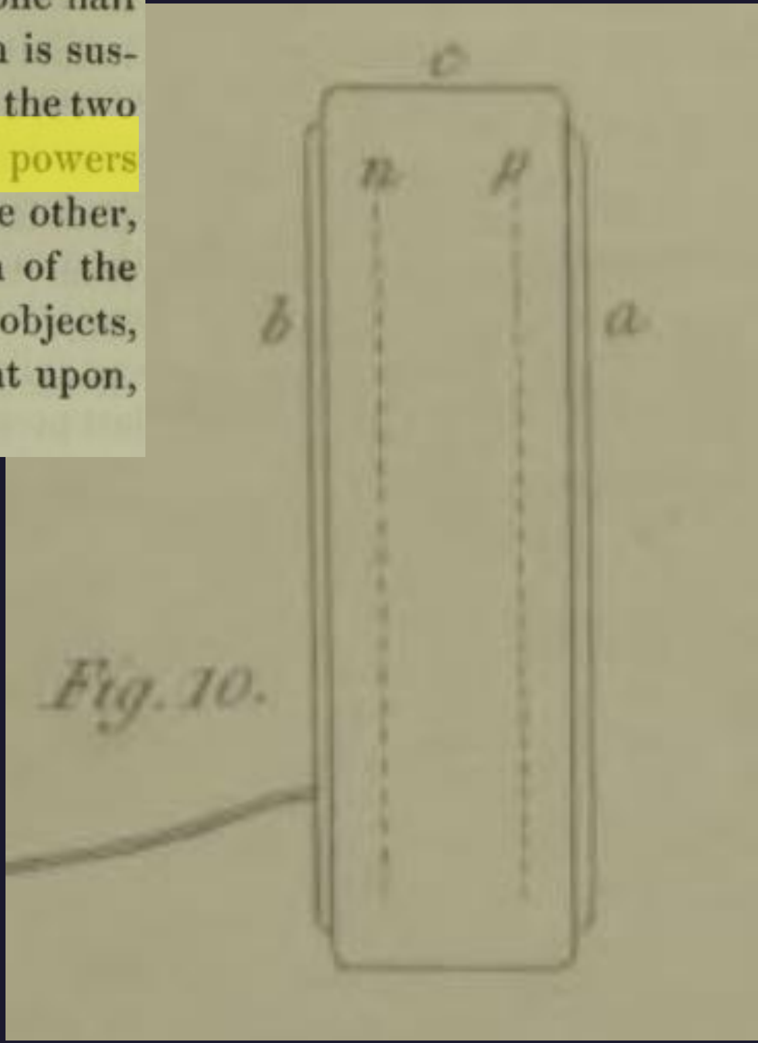
1252. I now proceed to examine the great question of specific inductive capacity, *i. e.* whether different dielectric bodies actually do possess any influence over the degree of induction which takes place through them. If any such difference should exist, it appeared to me not only of high importance in the further comprehension of the laws and results of induction, but dditional and very powerful argument for the theory I have red to put forth, that the whole depends upon a molecular in contradistinction to one at sensible distances.

estion may be stated thus : suppose A an electrified metal suspended in the air, and B and C two exactly plates, placed parallel to and on each side of A at equal and uninsulated ; A will then induce equally towards





1270. Admitting for the present the general fact sought to be proved; then 1·5, though it expresses the capacity of the apparatus containing the hemisphere of shell-lac, by no means expresses the relation of lac to air. The lac only occupies one half of the space *o, o,* of the apparatus containing it, through which the induction is sustained; the rest is filled with air, as in the other apparatus; and if the effect of the two upper halves of the globes be abstracted, then the comparison of the shell-lac powers in the lower half of the one, with the power of the air in the lower half of the other, will be as 2 : 1; and even this must be less than the truth, for the induction of the upper part of the apparatus, i. e. of the wire and ball B (fig. 1.) to external objects, must be the same in both, and considerably diminish the difference dependent upon, and really producible by, the influence of the shell-lac within.



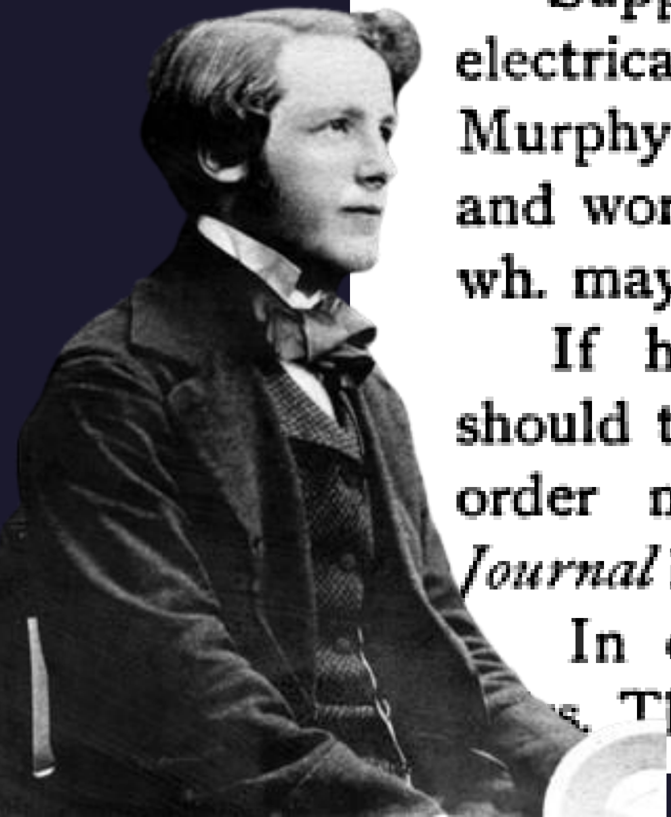

Dielectric Constant
(relative permittivity)

$$\epsilon_r = 2$$

Permittivity $= \epsilon_r \, \epsilon_0$

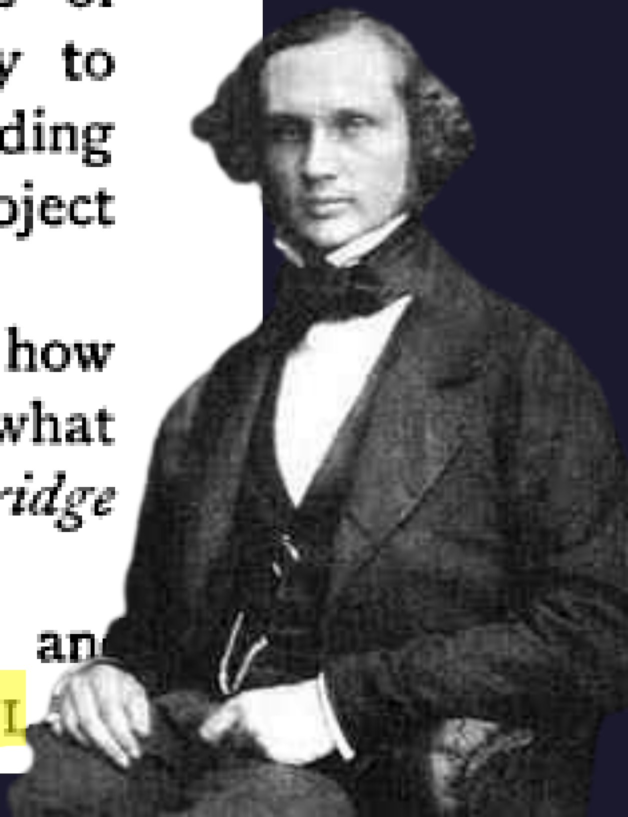
TRIN. COLL., *Feb.* 20, 1854.

DEAR THOMSON—Now that I have entered the unholy estate of bachelorhood I have begun to think of reading. This is very pleasant for some time among books of acknowledged merit wh. one has not read but ought to. But we have a strong tendency to return to physical subjects, and several of us here wish to attack Electricity.



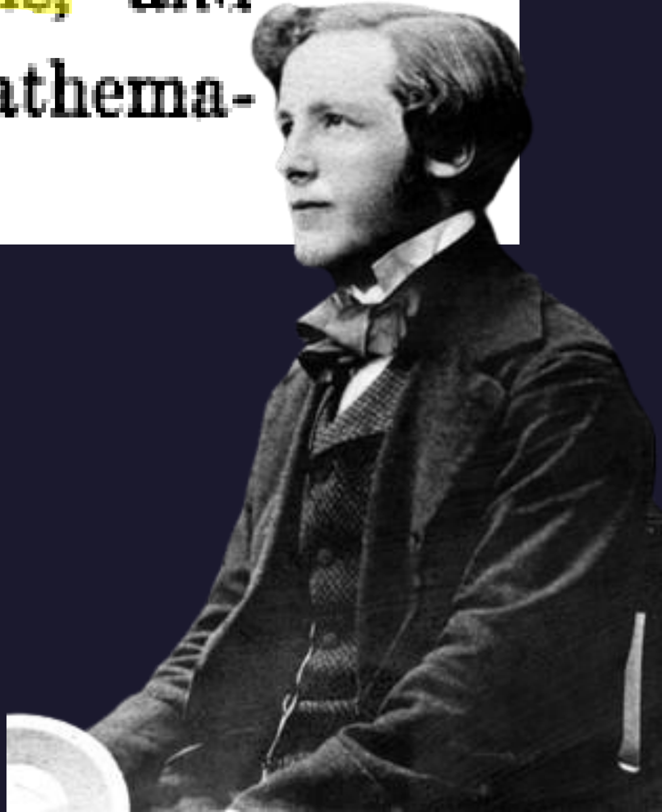
Suppose a man to have a popular knowledge of electrical show experiments and a little antipathy to Murphy's *Electricity*, how ought he to proceed in reading and working so as to get a little insight into the subject wh. may be of use in further reading?

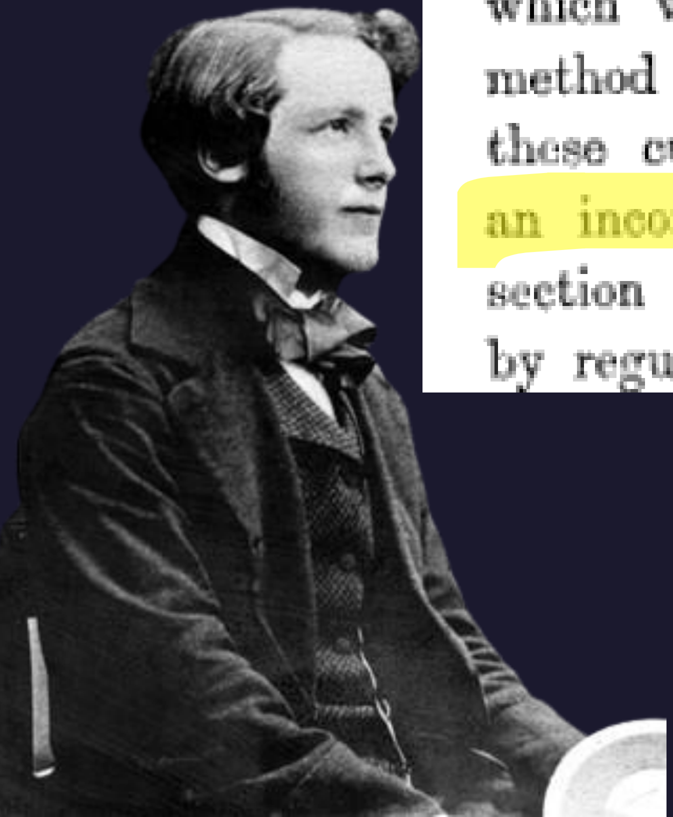
If he wished to read Ampère, Faraday, etc., how should they be arranged, and at what stage and in what order might he read your articles in the *Cambridge Journal*?



In conclusion, commend me to the Blackburns an
s. Thomson.—Yours truly, J. C. MAXWELL

As I proceeded with the study of Faraday, I perceived that his method of conceiving the phenomena was also a mathematical one, though not exhibited in the conventional form of mathematical symbols. I also found that these methods were capable of being expressed in the ordinary mathematical forms, and thus compared with those of the professed mathematicians.



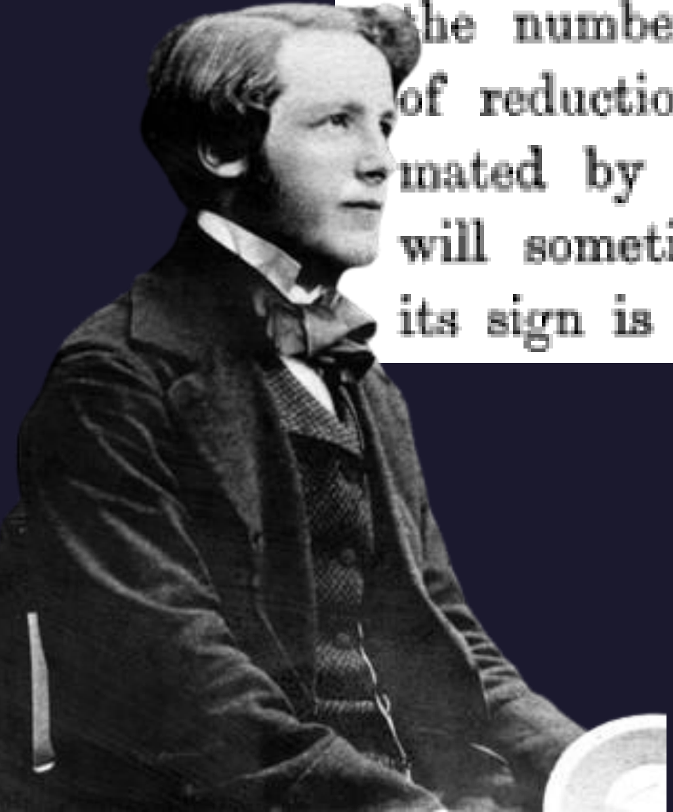


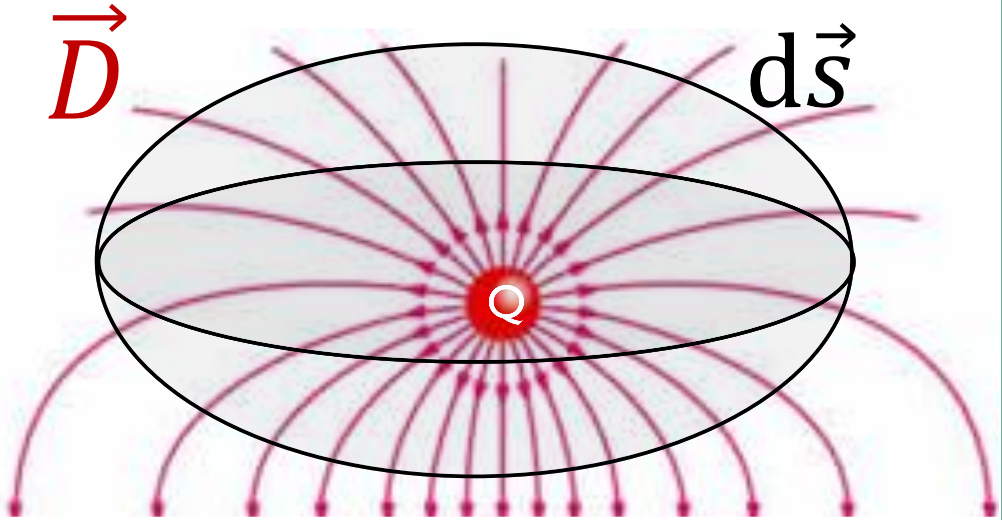
# VIII. *On Faraday's Lines of Force.*

[Read *Dec.* 10, 1855, and *Feb.* 11, 1856.]

We should thus obtain a geometrical model of the physical phenomena, which would tell us the *direction* of the force, but we should still require some method of indicating the *intensity* of the force at any point. If we consider these curves not as mere lines, but as fine tubes of variable section carrying an incompressible fluid, then, since the velocity of the fluid is inversely as the section of the tube, we may make the velocity vary according to any given law, by regulating the section of the tube, and in this way we might represent the

There is nothing self-contradictory in the conception of these sources where the fluid is created, and sinks where it is annihilated. The properties of the fluid are at our disposal, we have made it incompressible, and now we suppose it produced from nothing at certain points and reduced to nothing at others. The places of production will be called *sources*, and their numerical value will be the number of units of fluid which they produce in unit of time. The places of reduction will, for want of a better name, be called *sinks*, and will be estimated by the number of units of fluid absorbed in unit of time. Both places will sometimes be called sources, a source being understood to be a sink when its sign is negative.



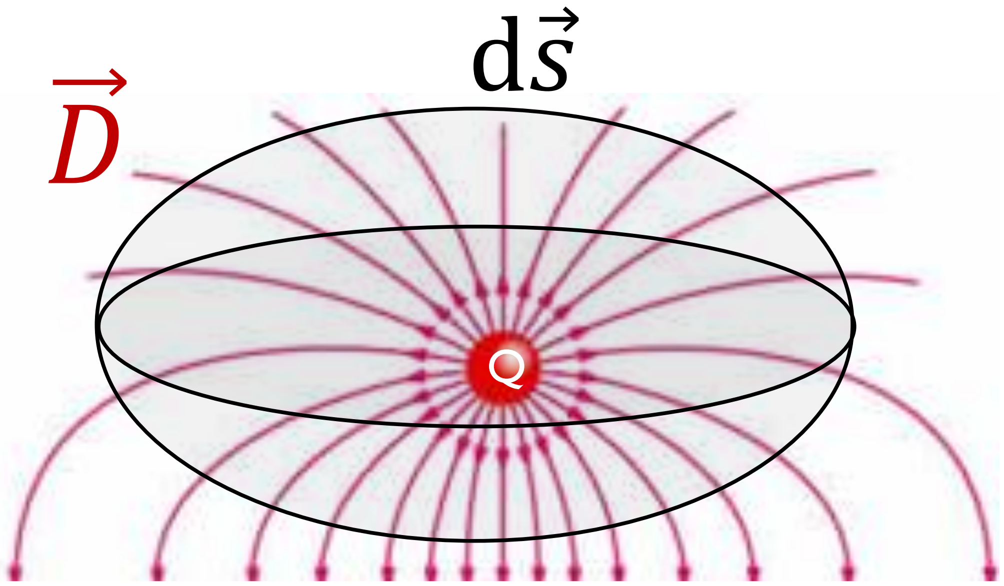



$$\oint \vec{D} \cdot \mathrm{d}\vec{s} = 4\pi Q$$

$$\oint \vec{D} \cdot \mathrm{d}\vec{s} = \iint D_x dydz + D_y dxdz + D_z dxdy$$

$$\oint \vec{D} \cdot \mathrm{d}\vec{s} = \iiint \left( \frac{dD_x}{dx} + \frac{dD_y}{dy} + \frac{dD_z}{dz} \right) dxdydz$$

$$\oint \vec{D} \cdot \mathrm{d}\vec{s} = \iiint (\vec{\nabla} \cdot \vec{D}) dxdydz$$




$$\oint \vec{D} \cdot d\vec{s} = 4\pi Q$$

$$Q = \iiint \rho \, dxdydz$$

$$\oint \vec{D} \cdot d\vec{s} = \iiint (\vec{\nabla} \cdot \vec{D}) dxdydz$$

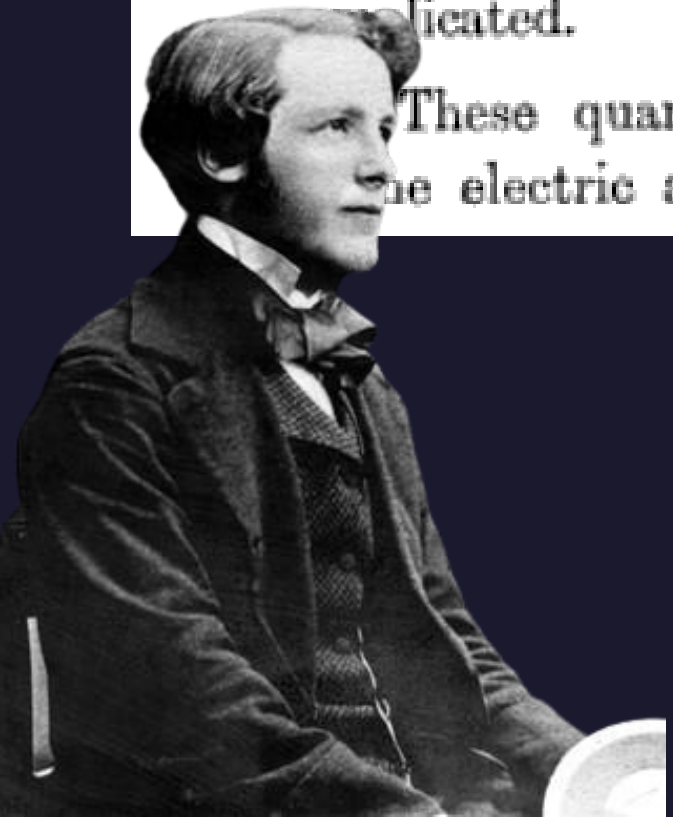
$$\vec{\nabla} \cdot \vec{D} = 4\pi\rho$$

Now the quantity of the current depends on the electro-motive force and on the resistance of the medium. If the resistance of the medium be uniform in all directions and equal to $k_2$,

$$\alpha_2 = k_2 a_2, \qquad \beta_2 = k_2 b_2, \qquad \gamma_2 = k_2 c_2 \quad \text{(B)},$$

but if the resistance be different in different directions, the law will be more …licated.

These quantities $\alpha_2$, $\beta_2$, $\gamma_2$ may be considered as representing the intensity …e electric action in the directions of $x$, $y$, $z$.



$$\vec{E} = k\,\vec{D}$$

Now the quantity of the current depends on the electro-motive force and on the resistance of the medium. If the resistance of the medium be uniform in all directions and equal to $k_2$,

$$\alpha_2 = k_2 a_2, \qquad \beta_2 = k_2 b_2, \qquad \gamma_2 = k_2 c_2 \quad \text{.........................} \text{(B)},$$

$$\vec{E} = k\,\vec{D}$$

but if the resistance be different in different directions, the law will be more complicated.

The total *quantity* of conduction through any surface is expressed by

$$\int e\,dS,$$

where

$$e = la + mb + nc,$$

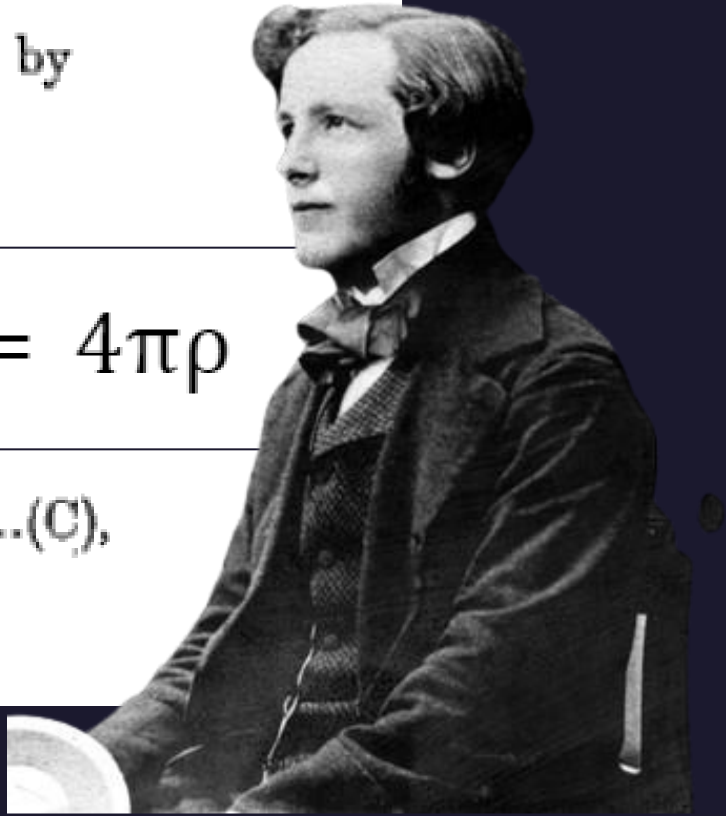
$l$, $m$, $n$ being the direction-cosines of the normal,

If we make

$$\vec{\nabla} \bullet \vec{D} = 4\pi\rho$$

$$\frac{da}{dx} + \frac{db}{dy} + \frac{dc}{dz} = 4\pi\rho \quad \text{...............................} \text{(C)},$$

$$\int e\,dS = 4\pi\iiint \rho\,dx\,dy\,dz,$$



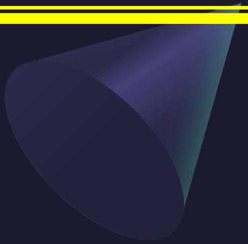
# In 1856 – Maxwell created Gauss's Law

$$\vec{E} = k\,\vec{D}$$

$$\vec{\nabla} \bullet \vec{D} = 4\pi\rho$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi k\rho \qquad \epsilon_0 = \frac{1}{4\pi k}$$

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_o$$



# Part 2

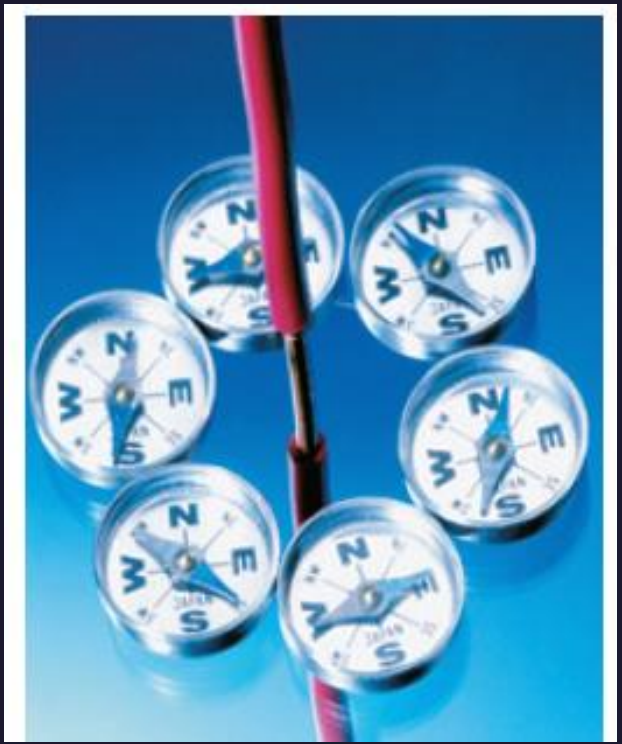
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

## Ampere's Law with Maxwell's addition

"Our physics would thus be no longer a collection of fragments on motion, on heat, on air, on light, on electricity, on magnetism, and who knows what else, but we would include the whole universe in one system."

Hans Christian Oersted (1803)

# May 1820



This action that M. OErsted discovered led me to look for the interaction of two electric currents, the action of the Earth on a current, and how the electricity might produce all phenomena presented by the magnets, looking for a distribution [of electric currents inside each magnet] similar to that of a conductor of electric current, with closed curves perpendicular to the axis of each magnet. These points of view, most of which have only recently been confirmed by experiment, were communicated to the *Académie* in its session of 18 September 1820.

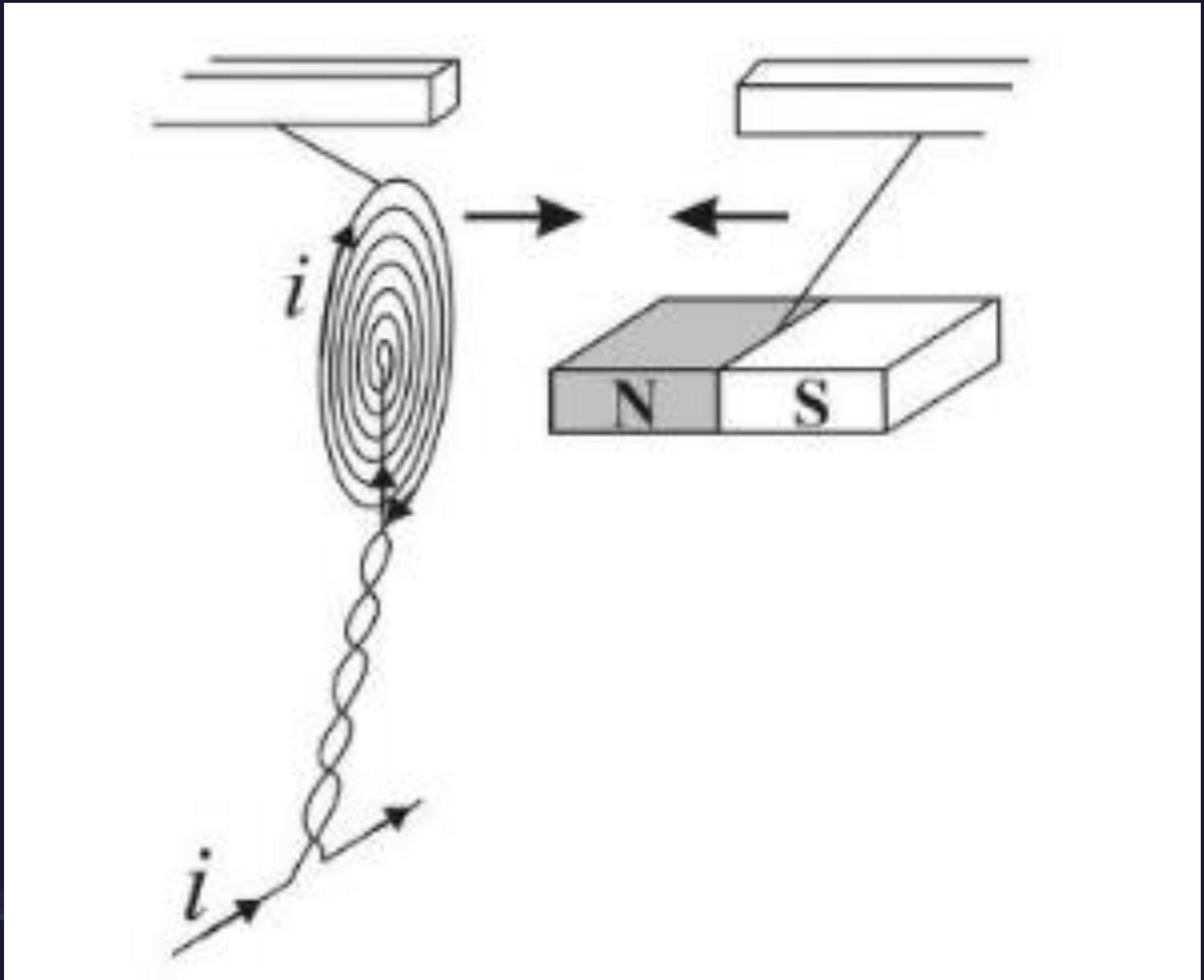
§ III. *De l'Action mutuelle entre un conducteur électrique et un aimant.*

C'est cette action découverte par M. OErsted, qui m'a conduit à reconnaître celle de deux courans électriques l'un sur l'autre, celle du globe terrestre sur un courant, et la manière dont l'électricité produisait tous les phénomènes que présentent les aimans, par une distribution semblable à celle qui a lieu dans le conducteur d'un courant électrique, suivant des courbes fermées perpendiculaires à l'axe de chaque aimant. Ces vues, dont la plus grande partie n'a été que plus tard confirmée par l'expérience, furent communiquées à l'Académie royale des Sciences, dans sa séance du 18 septembre 1820; je vais transcrire ce que je lus dans cette séance, sans autres changemens que la suppression des passages qui ne seraient qu'une répétition de ce que je viens de dire, et en particulier de ceux où je décrivais les appareils que je me proposais de faire construire; ils l'ont été depuis, et la plupart sont décrits dans les paragraphes précédens. On pourra, par ce moyen, se faire une idée plus juste de la marche que j'ai suivie dans mes recherches sur le sujet

énoncer sous la forme qui me paraît la plus simple et la plus générale.

Ces résultats consistent, d'une part, dans l'action directrice d'un de ces corps sur l'autre; de l'autre part, dans l'action attractive ou répulsive qui s'établit entre eux, suivant les circonstances.

*Action directrice.* Lorsqu'un aimant et un con[...] agissent l'un sur l'autre, et que l'un d'eux ét[...] l'autre ne peut que tourner dans un plan perp[...] à la plus courte distance du conducteur et[...] l'aimant, celui qui est mobile tend à se mou[...] nière que les directions du conducteur et de l[...] mant forment un angle droit, et que le pole d[...] qui regarde habituellement le nord soit à gau[...] qu'on appelle ordinairement *le courant galva[...]* nomination que j'ai cru devoir changer en [...] rant électrique, et le pole opposé à sa droit[...] que la ligne qui mesure la plus cou[...] ducteur et l'axe de l'aimant rencon[...] axe entre les deux poles. Pour co[...] toute la généralité dont il est susce[...] guer deux sortes de conducteurs :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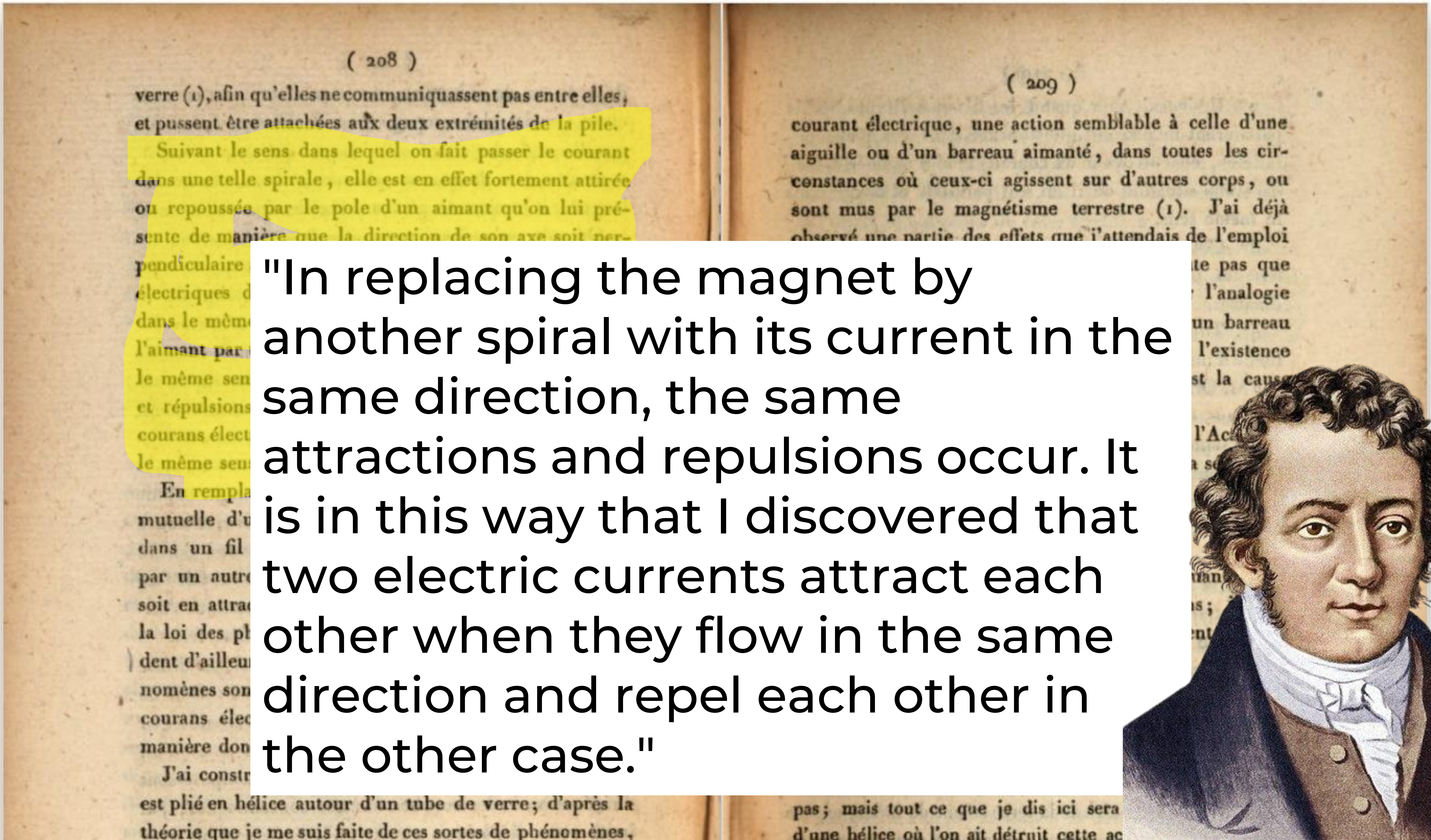

( 208 )

verre (1), afin qu'elles ne communiquassent pas entre elles, et pussent être attachées aux deux extrémités de la pile.

Suivant le sens dans lequel on fait passer le courant dans une telle spirale, elle est en effet fortement attirée ou repoussée par le pole d'un aimant qu'on lui présente de manière que la direction de son axe soit per-
pendiculaire
électriques d
dans le mêm
l'aimant par
le même sen
et répulsions
courans élect
le même sen

En rempla
mutuelle d'u
dans un fil
par un autr
soit en attra
la loi des ph
dent d'ailleu
nomènes son
courans élec
manière don

J'ai constr
est plié en hélice autour d'un tube de verre; d'après la théorie que je me suis faite de ces sortes de phénomènes,

( 209 )

courant électrique, une action semblable à celle d'une aiguille ou d'un barreau aimanté, dans toutes les circonstances où ceux-ci agissent sur d'autres corps, ou sont mus par le magnétisme terrestre (1). J'ai déjà observé une partie des effets que j'attendais de l'emploi
te pas que
l'analogie
un barreau
l'existence
st la caus
l'Ac

pas; mais tout ce que je dis ici sera
d'une hélice où l'on ait détruit cette ac

"In replacing the magnet by another spiral with its current in the same direction, the same attractions and repulsions occur. It is in this way that I discovered that two electric currents attract each other when they flow in the same direction and repel each other in the other case."

# THÉORIE

DES

## PHÉNOMÈNES ÉLECTRO-DYNAMIQUES,

UNIQUEMENT DÉDUITE DE L'EXPÉRIENCE,

PAR ANDRÉ-MARIE AMPÈRE,

DE L'ACADÉMIE ROYALE DES SCIENCES, DE LA SOCIÉTÉ PHILOMATIQUE, DE LA SOCIÉTÉ ROYALE D'ÉDIMBOURG, DE LA SOCIÉTÉ HELVÉTIENNE DES SCRUTATEURS DE LA NATURE, DE LA SOCIÉTÉ PHILOSOPHIQUE DE CAMBRIDGE, DE CELLE DE PHYSIQUE ET D'HISTOIRE NATURELLE DE GENÈVE, DE L'ACADÉMIE ROYALE DES SCIENCES ET BELLES-LETTRES DE BRUXELLES ET DE L'ACADÉMIE ROYALE DE LISBONNE, CHEVALIER DE LA LÉGION-D'HONNEUR, PROFESSEUR A L'ÉCOLE POLYTECHNIQUE ET AU COLLÉGE DE FRANCE.



A PARIS,

CHEZ MÉQUIGNON-MARVIS, LIBRAIRE-ÉDITEUR, RUE DU JARDINET, N° 13.

ET A BRUXELLES, AU DÉPÔT GÉNÉRAL DE LIBRAIRIE MÉDICALE FRANÇAISE.

NOVEMBRE 1826.

1826



en nommant $l$ la longueur des conducteurs, et quand ce rectangle devient un carré, on a $\frac{ii'}{\sqrt{2}}$ pour la valeur de la force; enfin, si l'on suppose l'un des conducteurs indéfini dans les deux sens, et que $l$ soit la longue de l'autre, les termes où $r_1', r_2', r_1'', r_2''$ se trouvent au dénominateur disparaîtront; on aura

$$r_2' + r_1'' - r_2'' - r_1' = 2l,$$

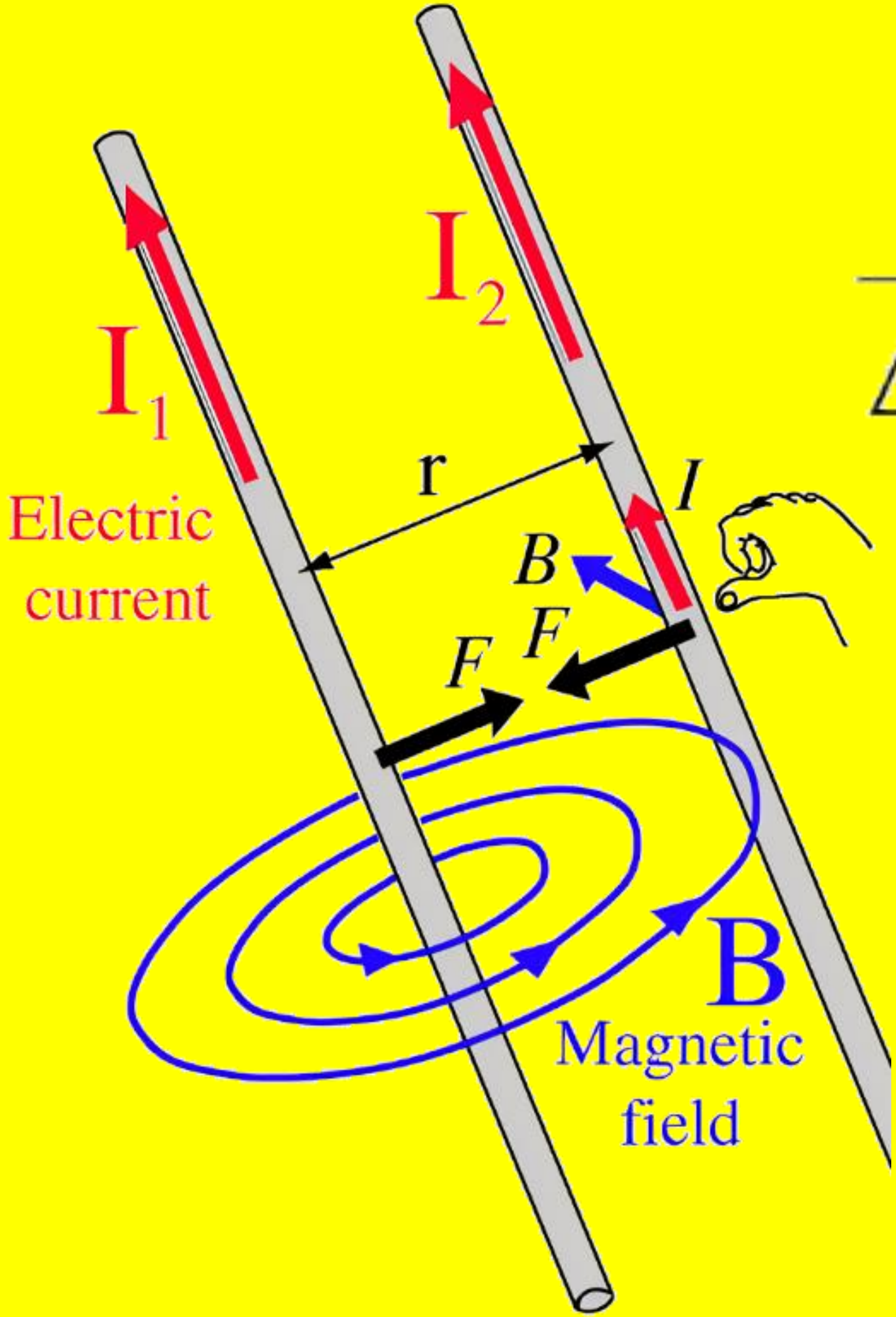
et l'expression de la force deviendra

$$\frac{ii'l}{a},$$

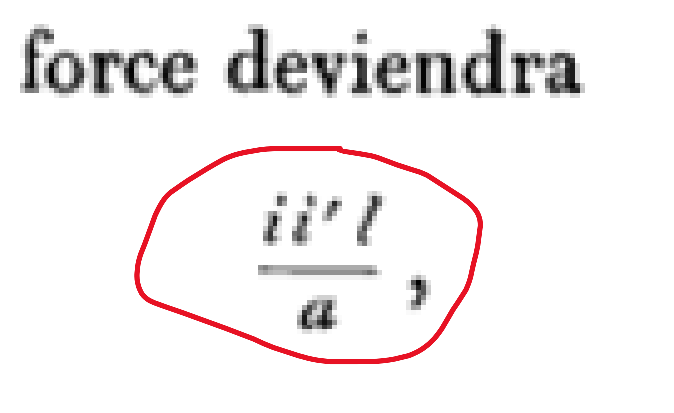
qui se réduit à $ii'$ quand la longueur $l$ est égale à la tance $a$.

Quant à l'action de deux courants parallèlement à rection de $s'$, elle peut s'obtenir quelle que soit l courant $s$. En effet la composante suivant à d$s'$

$$\frac{1}{2} ii' ds' d\left(\frac{\cos.^2 \beta}{r}\right),$$



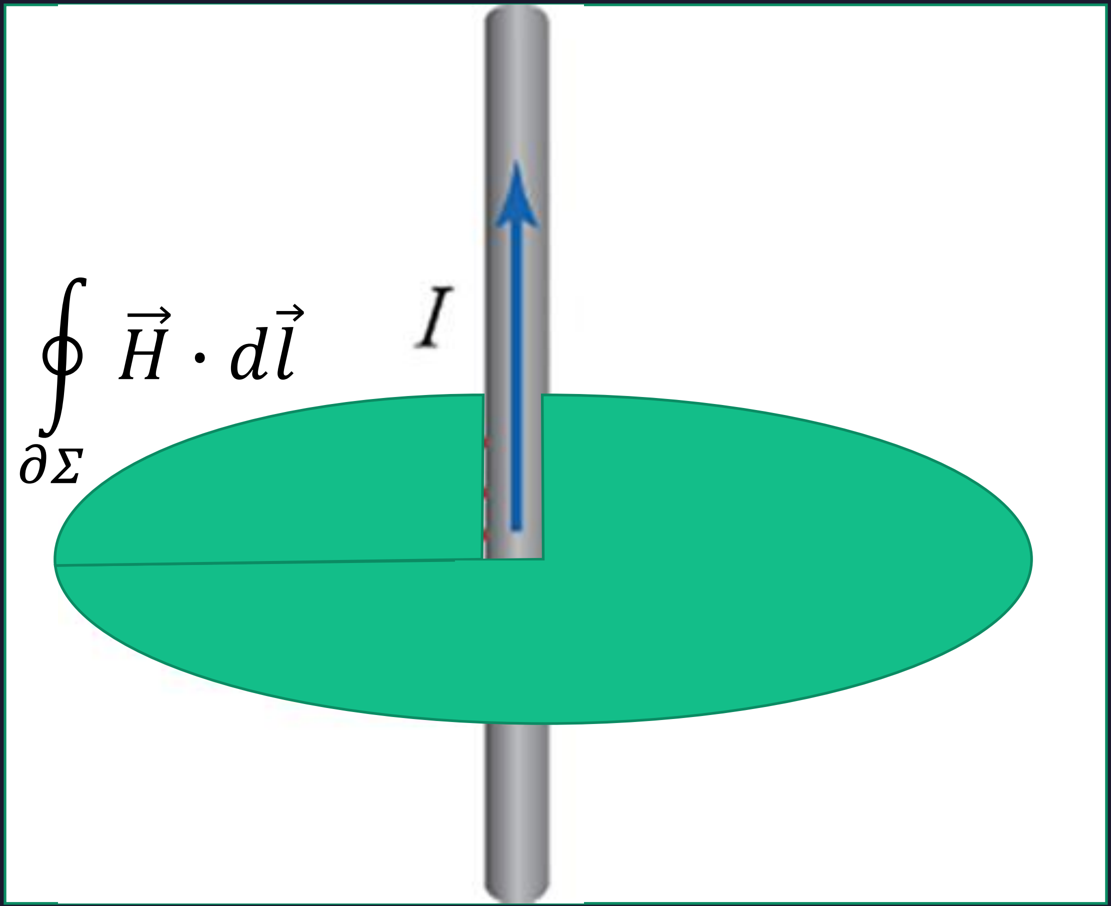

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$


force deviendra

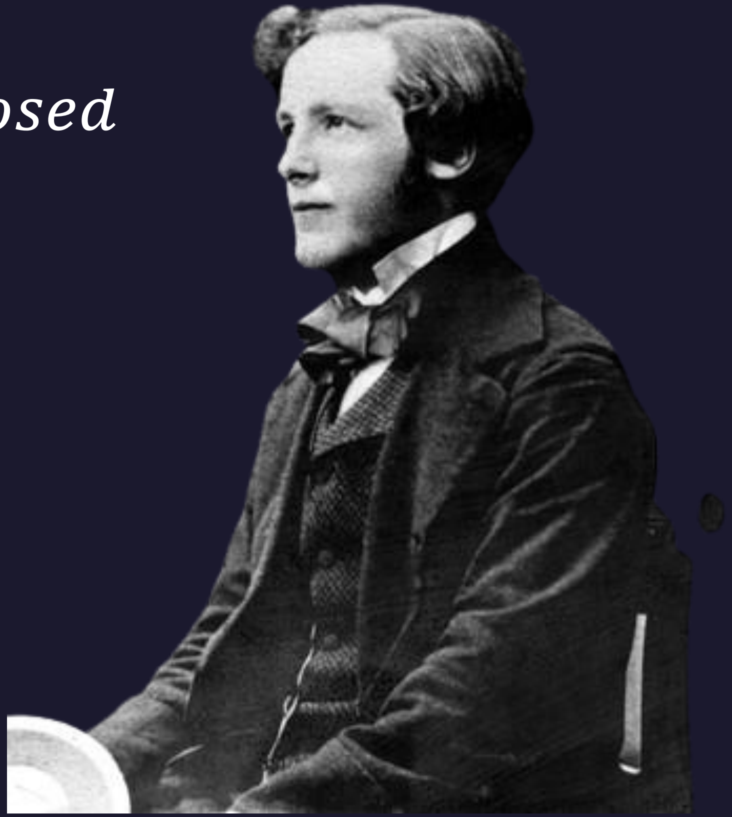
$$\frac{ii'l}{a},$$

194 ON FARADAY'S LINES OF FORCE.

The total intensity of magnetizing force in a closed curve passing through and embracing the closed current is constant, and may therefore be made a measure of the quantity of the current. As this intensity is independent of the

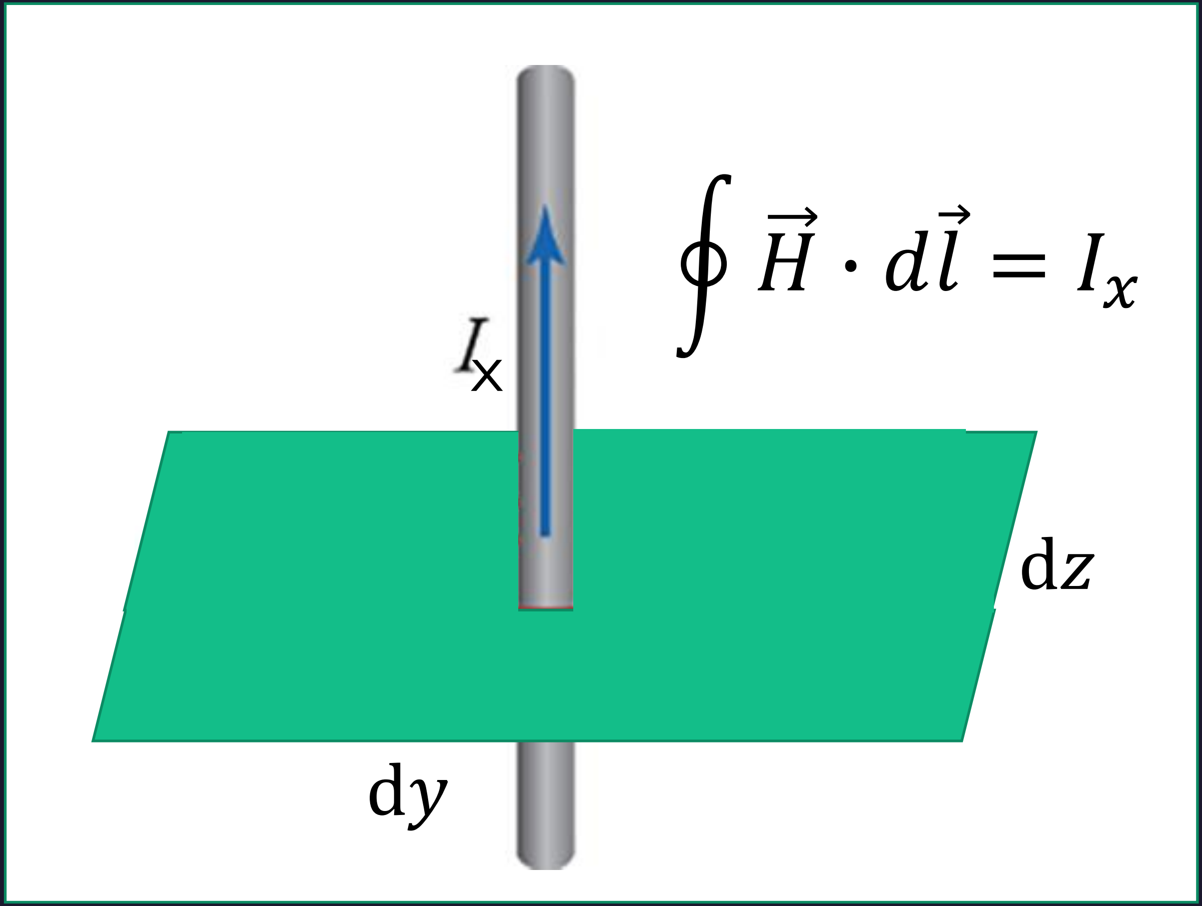



$$\oint_{\partial\Sigma} \vec{H} \cdot d\vec{l} = I_{inclosed}$$



passes through it, we may consider the elementary case of the current which flows through the elementary area *dydz*.

$$\oint \vec{H} \cdot d\vec{l} = \iint \left(\frac{dH_y}{dz} - \frac{dH_z}{dy}\right) dydz$$




$$I_x = \iint j_x dydz$$

$$j_x = \frac{dH_y}{dz} - \frac{dH_z}{dy}$$

Let the axis of $x$ point towards the west, $z$ towards the south, and $y$ upwards. Let $x, y, z$ be the coordinates of a point in the middle of the area $dydz$, then the total intensity measured round the four sides of the element is

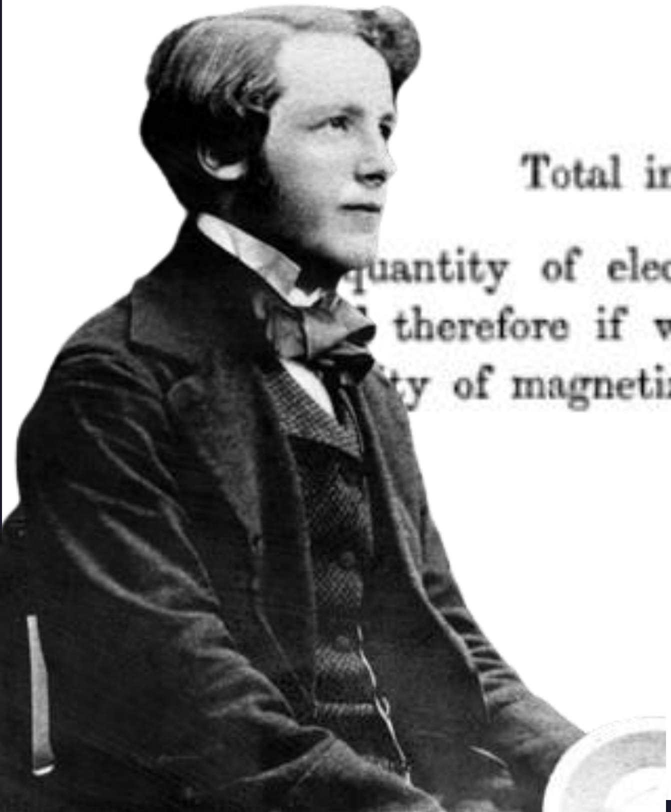
$$+\left(\beta_1 + \frac{d\beta_1}{dz}\frac{dz}{2}\right)dy,$$

$$-\left(\gamma_1 + \frac{d\gamma_1}{dy}\frac{dy}{2}\right)dz,$$

$$-\left(\beta_1 - \frac{d\beta_1}{dz}\frac{dz}{2}\right)dy,$$

$$+\left(\gamma_1 - \frac{d\gamma_1}{dy}\frac{dy}{2}\right)dz,$$

$$\text{Total intensity} = \left(\frac{d\beta_1}{dz} - \frac{d\gamma_1}{dy}\right)dy\,dz.$$



quantity of electricity conducted through the elementary area $dydz$ is

therefore if we define the measure of an electric current to be the

ty of magnetizing force in a closed curve embracing it, we shall have

$$a_2 = \frac{d\beta_1}{dz} - \frac{d\gamma_1}{dy},$$

$$b_2 = \frac{d\gamma_1}{dx} - \frac{d\alpha_1}{dz},$$

$$c_2 = \frac{d\alpha_1}{dy} - \frac{d\beta_1}{dx}.$$

$$\oint \vec{H} \cdot d\vec{l} = \iint \left(\frac{dH_y}{dz} - \frac{dH_z}{dy}\right) dydz$$

$$j_x = \frac{dH_z}{dy} - \frac{dH_y}{dz}$$

194 ON FARADAY'S LINES OF FORCE.

The quantity of electricity conducted through the elementary area $dydz$ is $a_2dydz$, and therefore if we define the measure of an electric current to be the total intensity of magnetizing force in a closed curve embracing it, we shall have

$$a_2 = \frac{d\beta_1}{dz} - \frac{d\gamma_1}{dy},$$

$$b_2 = \frac{d\gamma_1}{dx} - \frac{da_1}{dz},$$

$$c_2 = \frac{da_1}{dy} - \frac{d\beta_1}{dx}.$$

$$j_x = \frac{dH_z}{dy} - \frac{dH_y}{dz}$$

$$j_y = \frac{dH_z}{dx} - \frac{dH_x}{dz}$$

$$j_z = \frac{dH_x}{dy} - \frac{dH_y}{dx}$$

$$a_2 = \frac{d\beta_1}{dz} - \frac{d\gamma_1}{dy},$$

$$b_2 = \frac{d\gamma_1}{dx} - \frac{d\alpha_1}{dz},$$

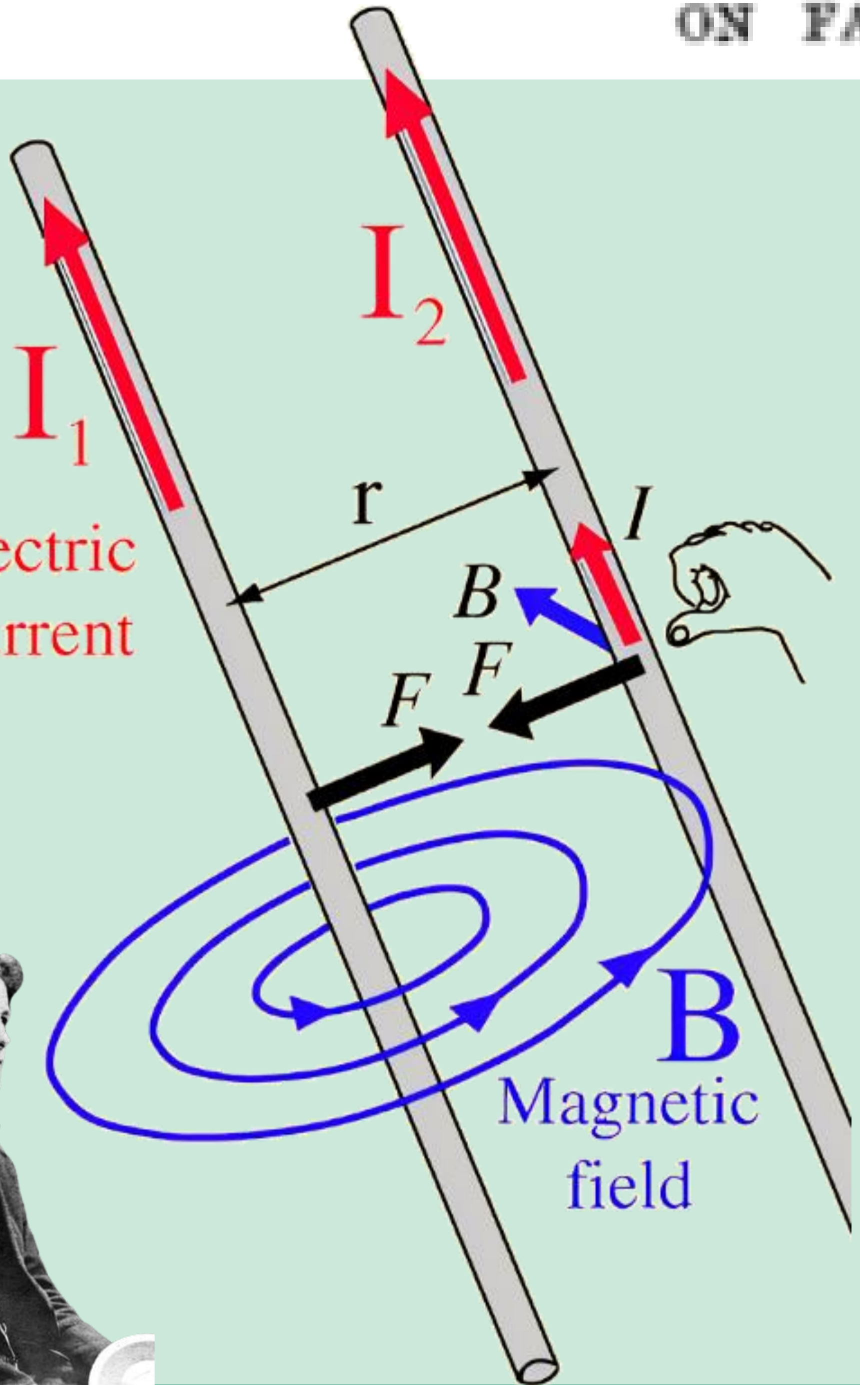
$$c_2 = \frac{d\alpha_1}{dy} - \frac{d\beta_1}{dx}.$$

$$j_x = \frac{dH_z}{dy} - \frac{dH_y}{dz}$$

$$j_y = \frac{dH_z}{dx} - \frac{dH_x}{dz}$$

$$j_z = \frac{dH_x}{dy} - \frac{dH_y}{dx}$$

$$\vec{j} = \vec{\nabla} \times \vec{H}$$




$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$\vec{j} = \vec{\nabla} \times \vec{H}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$$

VII.—*On the Mechanical Action of Heat, especially in Gases and Vapours.* By WILLIAM JOHN MACQUORN RANKINE, Civil Engineer, F.R.S.E., F.R.S.S.A., &c.

(Read 4th February 1850.)

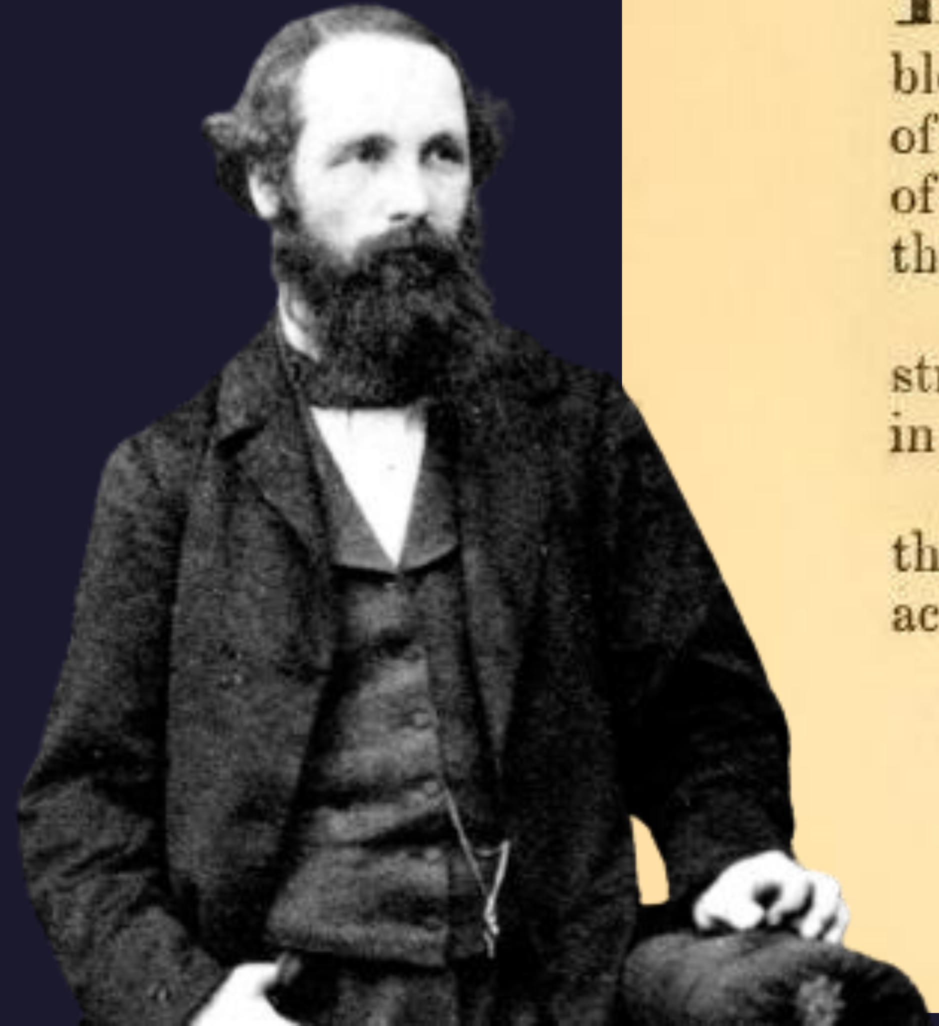
## INTRODUCTION.

### SUMMARY OF THE PRINCIPLES OF THE HYPOTHESIS OF MOLECULAR VORTICES, AND ITS APPLICATION TO THE THEORY OF TEMPERATURE, ELASTICITY, AND REAL SPECIFIC HEAT.

The fundamental suppositions are the following:—

*First,—That each atom of matter consists of a nucleus, or central physical point, enveloped by an elastic atmosphere, which is retained in its position by forces attractive towards the nucleus or centre.*

Suppositions similar to this have been brought forward by FRANKLIN, ÆPINUS, MOSSOTTI, and others. They have in general, however, conceived the atmosphere of each nucleus to be of variable mass. I have treated it, on the contrary, as an essential part of the atom. I have left the question indeterminate, whether the nucleus is a small body of a character distinct from that of the atmosphere, or merely a portion of the atmosphere in a highly condensed state, owing to the mutual attraction of its parts.



III. *On Physical Lines of Force.* By J. C. Maxwell, *F.R.S., Professor of Natural Philosophy in King's College, London**.

PART III.—*The Theory of Molecular Vortices applied to Statical Electricity.*

IN the first part of this paper† I have shown how the forces acting between magnets, electric currents, and matter capable of magnetic induction may be accounted for on the hypothesis of the magnetic field being occupied with innumerable vortices of revolving matter, their axes coinciding with the direction of the magnetic force at every point of the field.

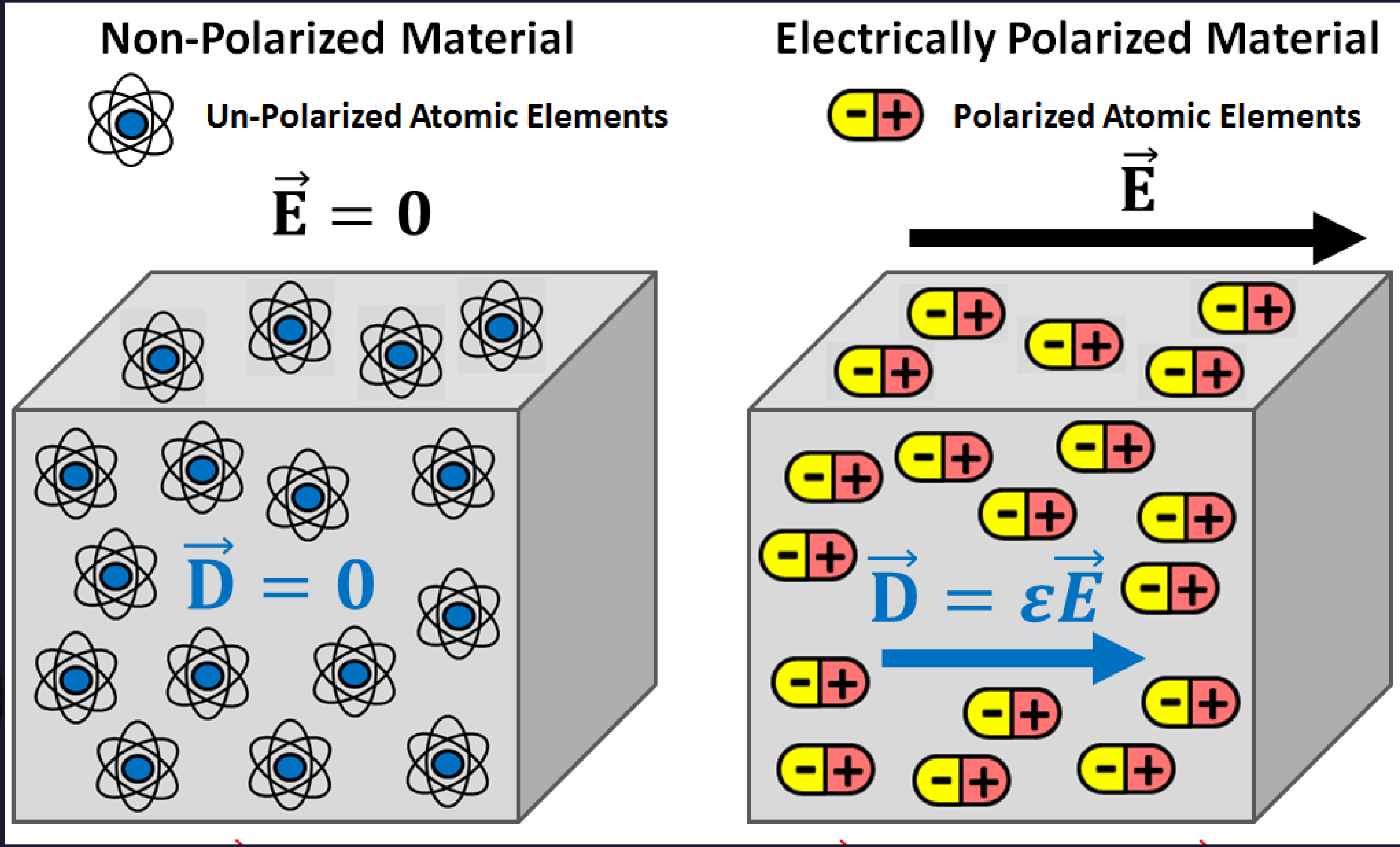
The centrifugal force of these vortices produces pressures distributed in such a way that the final effect is a force identical in direction and magnitude with that which we observe.

In the second part‡ I described the mechanism by which these rotations may be made to coexist, and to be distributed according to the known laws of magnetic lines of force.

* Communicated by the Author.
† Phil. Mag. March 1861.
Phil. Mag. April and May 1861.

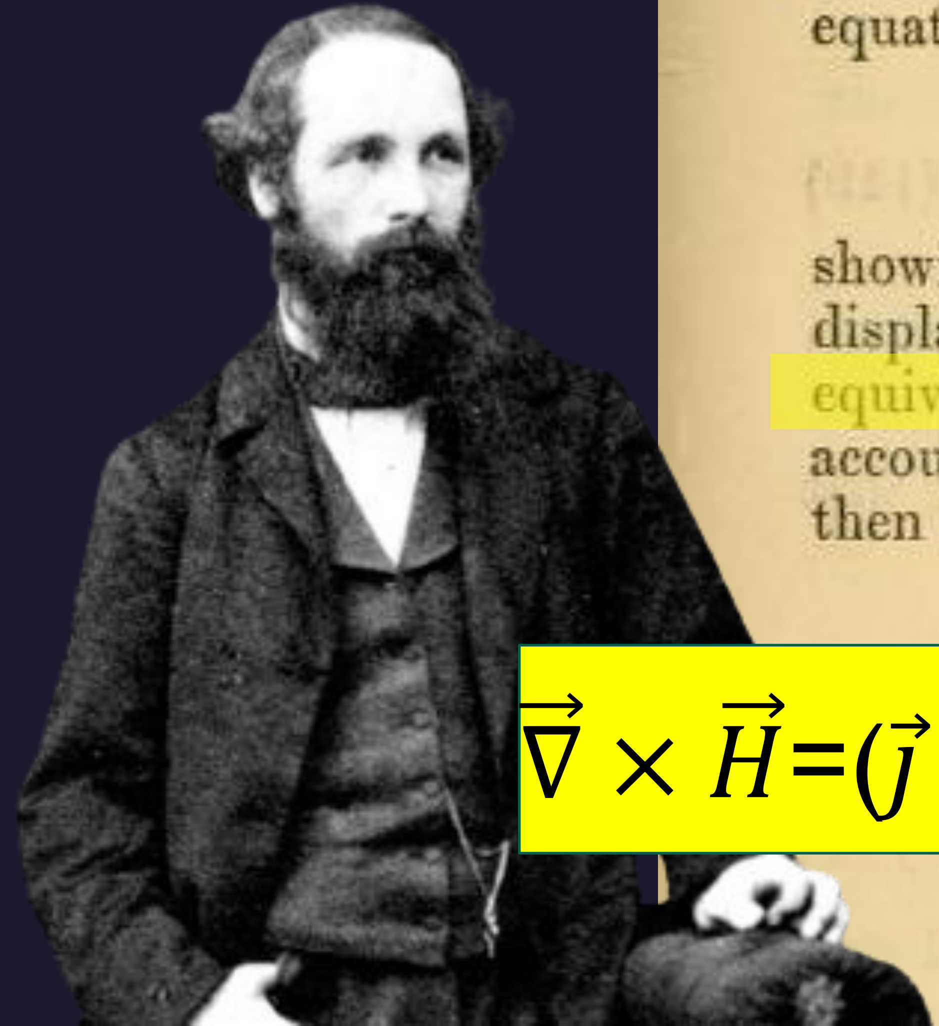
Received November 30,—Read December 21, 1837.

I am about to adduce in favour of my view, that electric induction is an action of the contiguous particles of the insulating medium or *dielectric**.



In a dielectric under induction, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively, and the other negatively electrical, but that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another.

The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction. This displacement does not amount to a current, because when it has attained a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or negative direction, accordi as the displacement is increasing or diminishing. The am of the displacement depends on the nature of the body, ar



$$\vec{\nabla} \times \vec{H} = (\vec{J} + \frac{d\vec{D}}{dt})$$



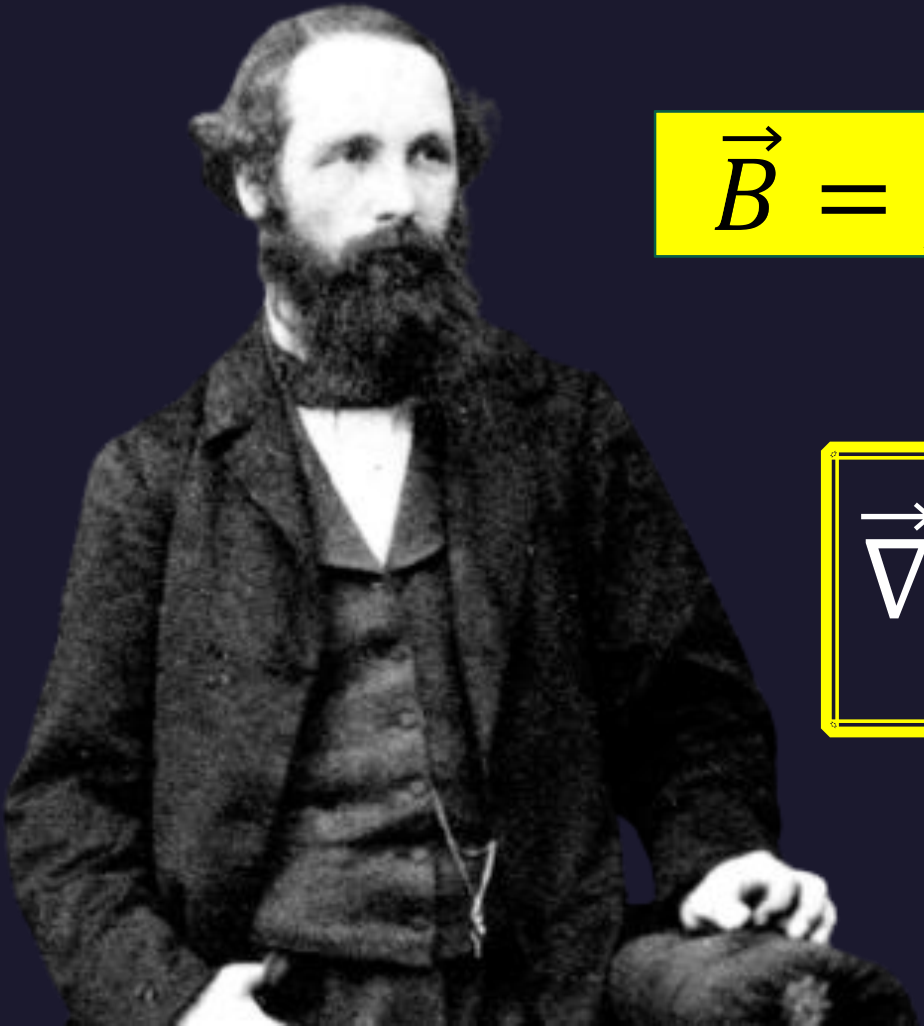
*Prop.* XIV.—To correct the equations (9)† of electric currents for the effect due to the elasticity of the medium.

We have seen that electromotive force and electric displacement are connected by equation (105). Differentiating this equation with respect to $t$, we find

$$\frac{dR}{dt} = -4\pi E^2 \frac{dh}{dt}, \quad . \quad . \quad . \quad . \quad . \quad (111)$$

showing that when the electromotive force varies, the electric displacement also varies. But a variation of displacement is equivalent to a current, and this current must be taken into account in equations (9) and added to $r$. The three equations then become

$$\left.\begin{aligned} p &= \frac{1}{4\pi}\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz} - \frac{1}{E^2}\frac{dP}{dt}\right), \\ q &= \frac{1}{4\pi}\left(\frac{d\alpha}{dz} - \frac{d\gamma}{dx} - \frac{1}{E^2}\frac{dQ}{dt}\right), \\ r &= \frac{1}{4\pi}\left(\frac{d\beta}{dx} - \frac{d\alpha}{dy} - \frac{1}{E^2}\frac{dR}{dt}\right), \end{aligned}\right\} \quad . \quad . \quad (112)$$



$$\vec{\nabla} \times \vec{H} = (\vec{J} + \frac{d\vec{D}}{dt})$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$$

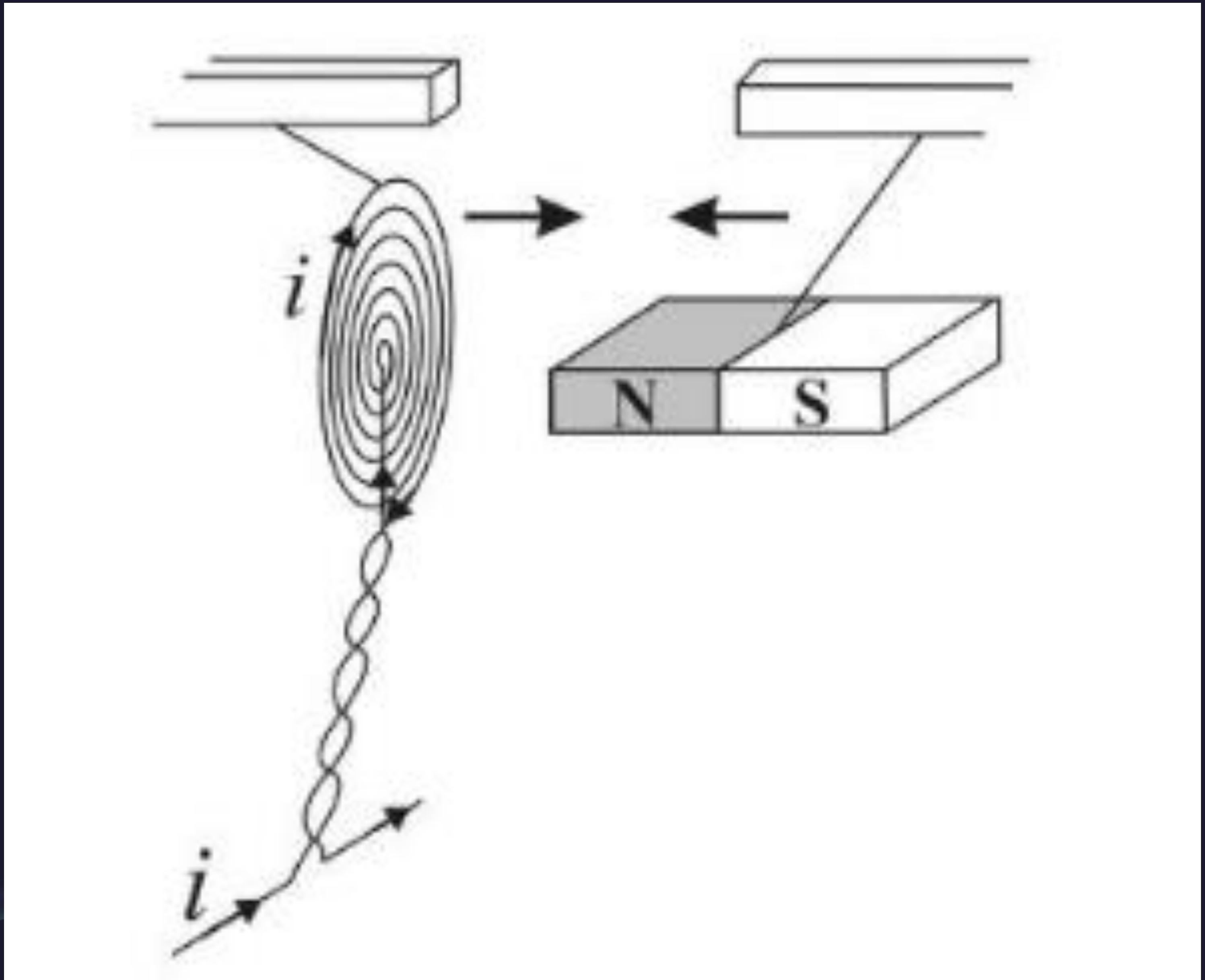
# Part 3

Faraday's Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

+ Gauss's Law for Magnets

$$\nabla \cdot \mathbf{B} = 0$$




# 1820

"When the conducting wire which forms the helix … by enclosing it inside a glass tube … [the wires acted] perfectly similar to the action of a magnet"



mutuelle d'un des poles d'un aimant et d'un courant dans un fil métallique plié en spirale, cette spirale par un autre aimant, on a encore les mêmes effets, soit en attraction, soit en répulsion, conformément à la loi des phénomènes connus de l'aimant; il est évident d'ailleurs que toutes les circonstances de ces phénomènes sont une suite nécessaire de la disposition des courans électriques dont ils se composent, d'après la manière dont ceux-ci s'attirent et se repoussent.

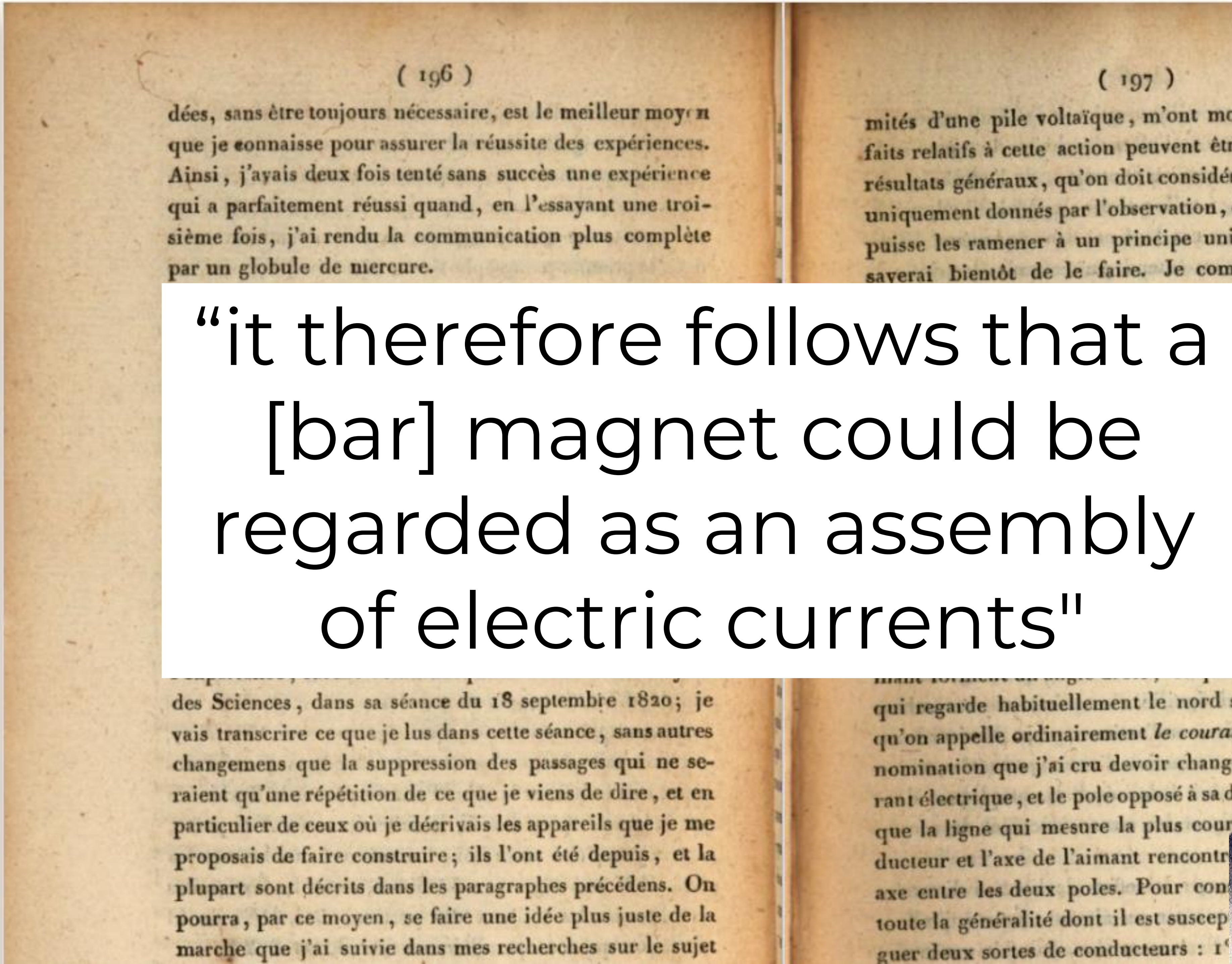
J'ai construit un autre appareil où le fil conducteur est plié en hélice autour d'un tube de verre; d'après la théorie que je me suis faite de ces sortes de phénomènes,



une action semblable à celle d'une
rreau aimanté, dans toutes les cir-
-ci agissent sur d'autres corps, ou
agnétisme terrestre (1). J'ai déjà
les effets que j'attendais de l'emploi
é en hélice, et je ne doute pas que
expériences fondées sur l'analogie
entre cet instrument et un barreau
btiendra de preuves que l'existence
ques dans les aimans
phénomènes magnét
r la lecture que j
de transcrire, qu
terminai cette l
où je déduisais, des faits qui y ét
clusions suivantes :

1°. Deux courans électriques s'at
meuvent parallèlement dans le mê
poussent quand ils se meuvent pa
contraire.

(1) Quand j'écrivais cela
tion exercée par les spires
axe, et je croyais qu'on p
pas; mais tout ce que je
d'une hélice où l'on ait dét


( 196 )

dées, sans être toujours nécessaire, est le meilleur moyen que je connaisse pour assurer la réussite des expériences. Ainsi, j'avais deux fois tenté sans succès une expérience qui a parfaitement réussi quand, en l'essayant une troisième fois, j'ai rendu la communication plus complète par un globule de mercure.

des Sciences, dans sa séance du 18 septembre 1820; je vais transcrire ce que je lus dans cette séance, sans autres changemens que la suppression des passages qui ne seraient qu'une répétition de ce que je viens de dire, et en particulier de ceux où je décrivais les appareils que je me proposais de faire construire; ils l'ont été depuis, et la plupart sont décrits dans les paragraphes précédens. On pourra, par ce moyen, se faire une idée plus juste de la marche que j'ai suivie dans mes recherches sur le sujet

( 197 )

mités d'une pile voltaïque, m'ont montré que tous les faits relatifs à cette action peuvent être ramenés à deux résultats généraux, qu'on doit considérer d'abord comme uniquement donnés par l'observation, en attendant qu'on puisse les ramener à un principe unique, comme j'essaverai bientôt de le faire. Je commencerai par les

la plus simple et

rt, dans l'action

; de l'autre part,

qui s'établit entre

qui regarde habituellement le nord soit à gauch

qu'on appelle ordinairement *le courant galvani*

nomination que j'ai cru devoir changer en cel

rant électrique, et le pole opposé à sa droite

que la ligne qui mesure la plus court

ducteur et l'axe de l'aimant rencontr

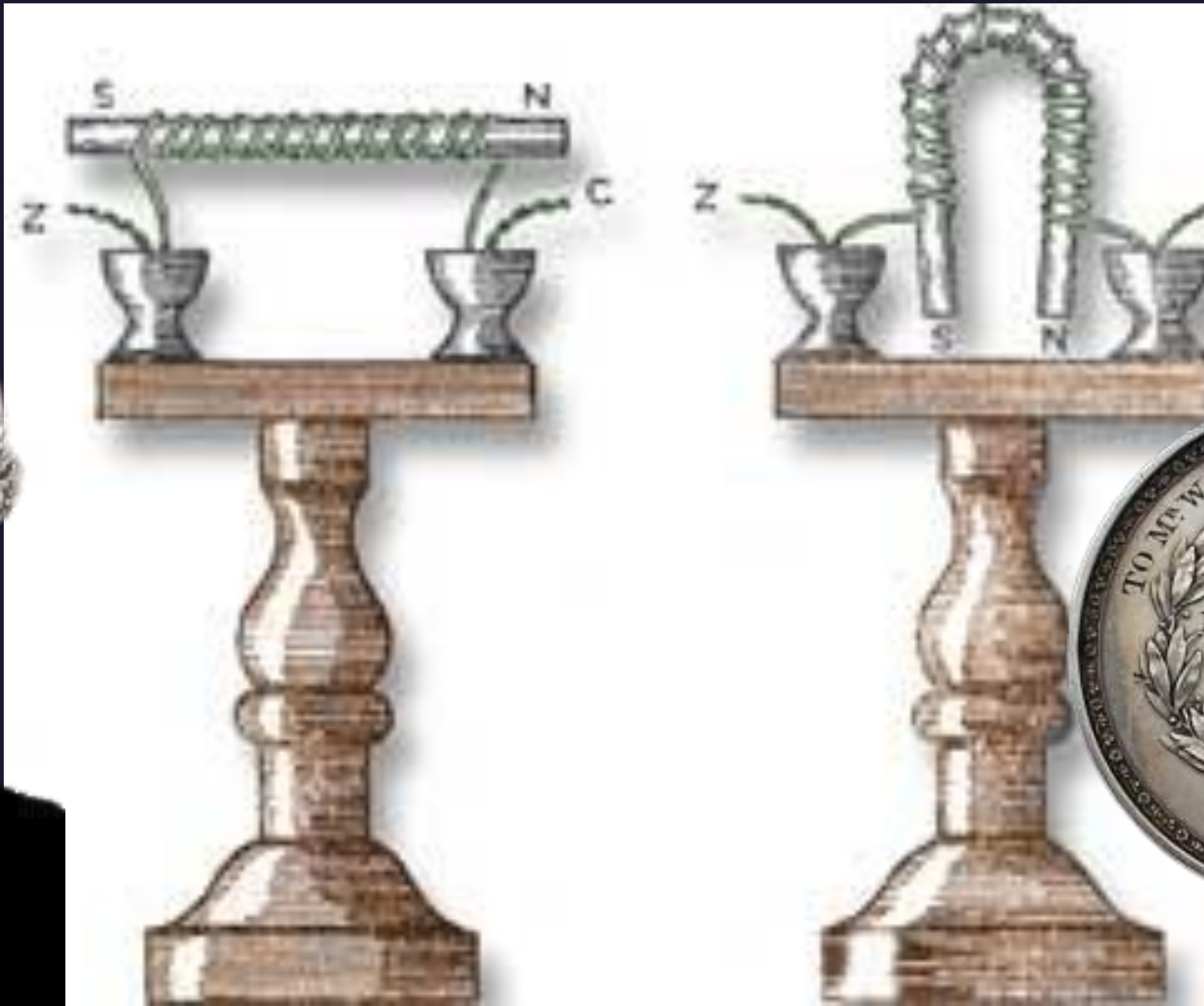
axe entre les deux poles. Pour con

toute la généralité dont il est suscep

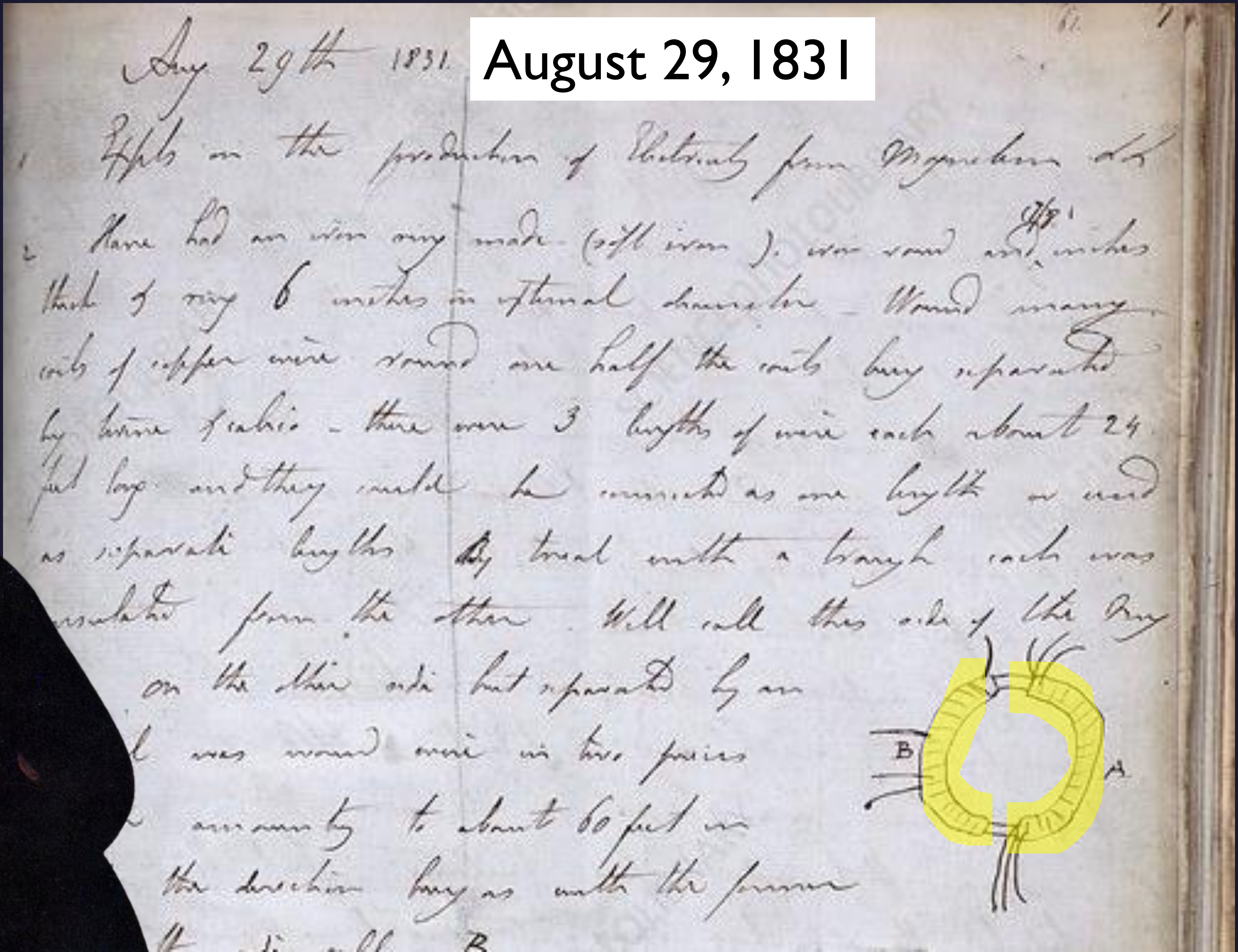
guer deux sortes de conducteurs : 1°



# 1825 William Sturgeon




August 29, 1831



Aug 29th 1831.

1 Expts on the production of Electricity from Magnetism &c

2 Have had an iron ring made (soft iron), iron round and 7/8 inches thick & ring 6 inches in external diameter. Wound many coils of copper wire round one half the coils being separated by twine & calico — there were 3 lengths of wire each about 24 feet long and they could be connected as one length or used as separate lengths. By trial with a trough each was insulated from the other. Will call this side of the ring A

on the other side but separated by an

l was wound wire in two pieces

amounting to about 60 feet in

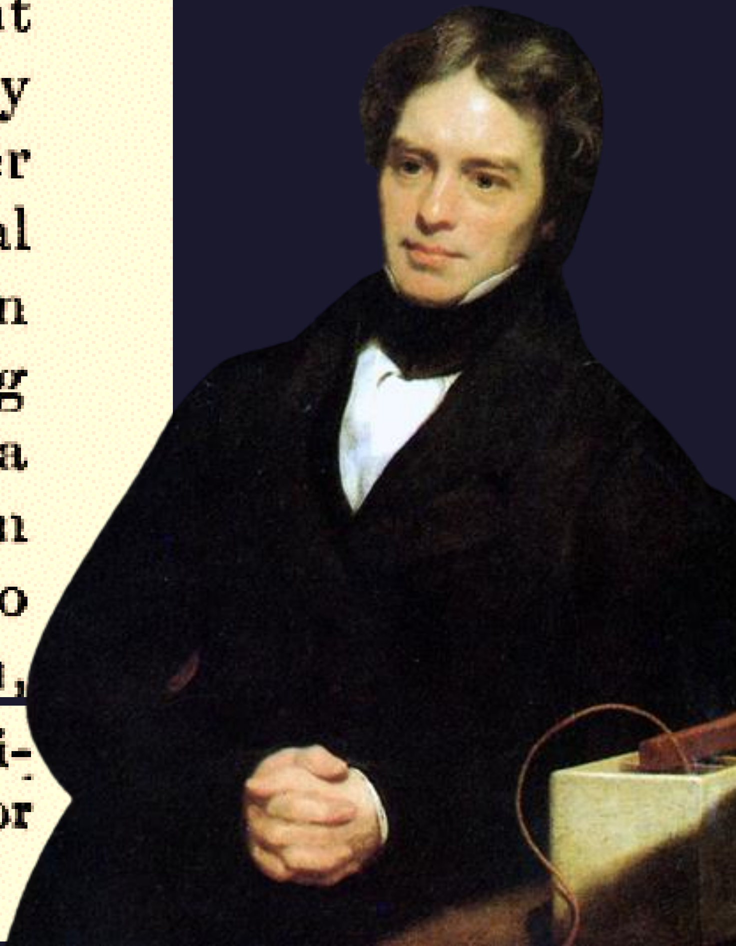
the direction being as with the former

B

§ 1. *On the Induction of Electric Currents.* § 2. *On the Evolution of Electricity from Magnetism.* § 3. *On a new Electrical Condition of Matter.* § 4. *On* Arago's *Magnetic Phenomena.*

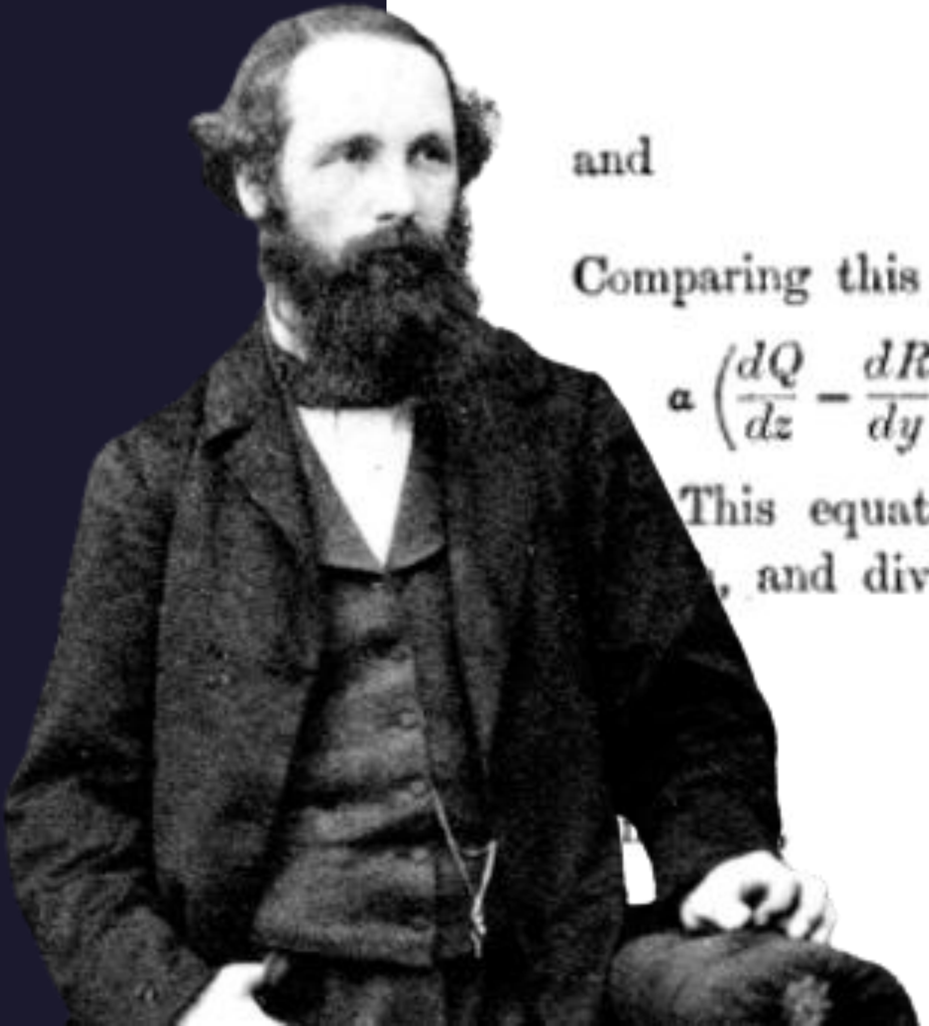
[Read November 24, 1831.]

114. The relation which holds between the magnetic pole, the moving wire or metal, and the direction of the current evolved, *i. e. the law* which governs the evolution of electricity by magneto-electric induction, is very simple, although rather difficult to express. If in fig. 24. PN represent a horizontal wire passing by a marked magnetic pole, so that the direction of its motion shall coincide with the curved line proceeding from below upwards; or if its motion parallel to itself be in a line tangential to the curved line, but in the general direction of the arrows; or if it pass the pole in other directions, but so as to cut the magnetic curves * in the same general direction,



* By magnetic curves I mean the lines of magnetic forces, however modified by the juxtaposition of poles, which would be depicted by iron filings; or those to which a very small magnetic needle would form a tangent.

1862





PROP. VIII.—To find the relations between the alterations of motion of the vortices, and the forces $P, Q, R$ which they exert on the layer of particles between them.

Let $V$ be the volume of a vortex, then by (46) its energy is

$$E=\frac{1}{8\pi}\mu(\alpha^2+\beta^2+\gamma^2)V \quad \ldots\ldots\ldots\ldots (51),$$

and

$$\frac{dE}{dt}=\frac{1}{4\pi}\mu V\left(\alpha\frac{d\alpha}{dt}+\beta\frac{d\beta}{dt}+\gamma\frac{d\gamma}{dt}\right) \quad \ldots\ldots\ldots\ldots (52).$$

Comparing this value with that given in equation (50), we find

$$\alpha\left(\frac{dQ}{dz}-\frac{dR}{dy}-\mu\frac{d\alpha}{dt}\right)+\beta\left(\frac{dR}{dx}-\frac{dP}{dz}-\mu\frac{d\beta}{dt}\right)+\gamma\left(\frac{dP}{dy}-\frac{dQ}{dx}-\mu\frac{d\gamma}{dt}\right)=0 \ldots\ldots (53).$$

This equation being true for all values of $\alpha$, $\beta$, and $\gamma$, first let $\beta$ and $\gamma$ , and divide by $\alpha$. We find

$$\left.\begin{aligned}\frac{dQ}{dz}-\frac{dR}{dy}&=\mu\frac{d\alpha}{dt}\\ \frac{dR}{dx}-\frac{dP}{dz}&=\mu\frac{d\beta}{dt}\\ \frac{dP}{dy}-\frac{dQ}{dx}&=\mu\frac{d\gamma}{dt}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (54).$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{d\vec{H}}{dt}$$

1862



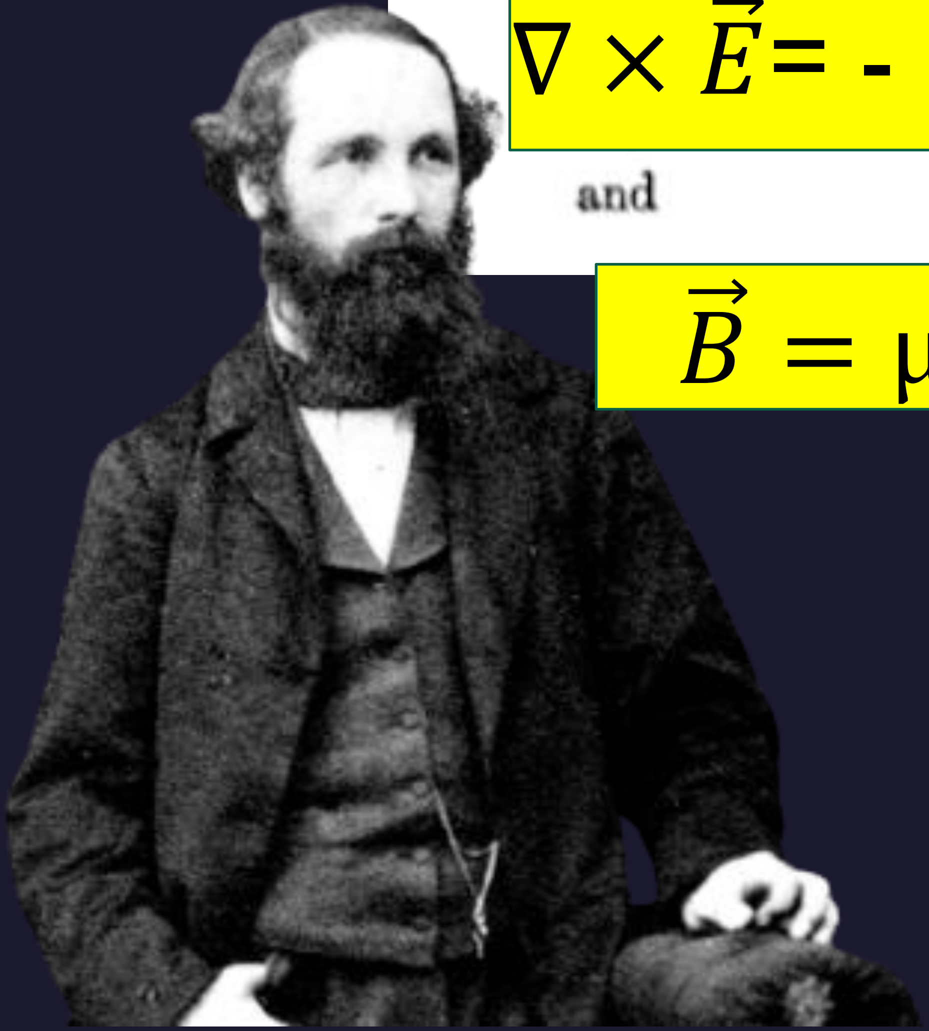
This equation being true for all values of $\alpha$, $\beta$, and $\gamma$, first let $\beta$ and $\gamma$ vanish, and divide by $\alpha$. We find

$$\left.\begin{aligned}\frac{dQ}{dz}-\frac{dR}{dy}&=\mu\frac{d\alpha}{dt}\\ \frac{dR}{dx}-\frac{dP}{dz}&=\mu\frac{d\beta}{dt}\\ \frac{dP}{dy}-\frac{dQ}{dx}&=\mu\frac{d\gamma}{dt}\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54).$$

and

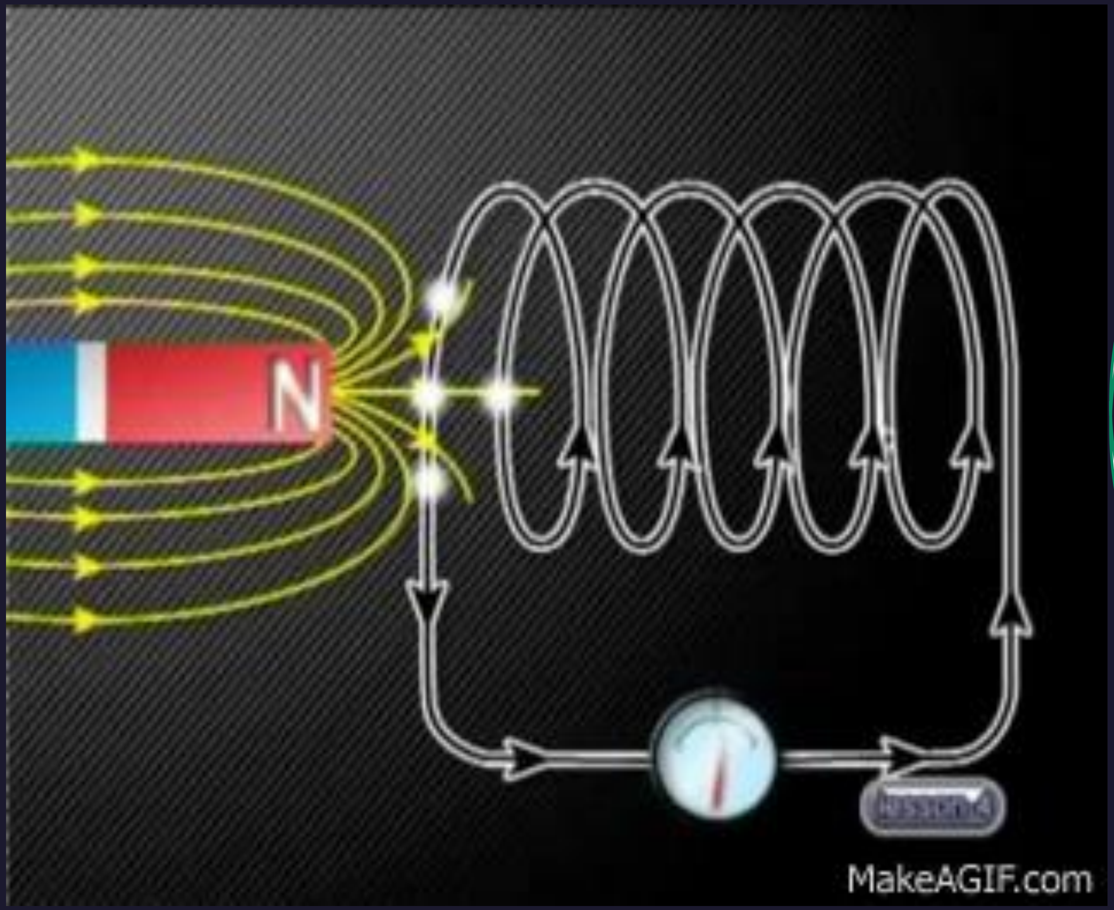
$$\nabla \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$$

$$\vec{B} = \mu\vec{H}$$



$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$




*curling E field*

$$\nabla \times \vec{E}$$

*changing B field* $-\dfrac{d\vec{B}}{dt}$

1862

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \cdot \frac{d\vec{B}}{dt}$$

0

$$0 = -\vec{\nabla} \cdot \frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

1862



Let us then find three quantities $F$, $G$, $H$ from the equations

$$\vec{\nabla} \times \vec{E} = -\mu \frac{d\vec{H}}{dt}$$

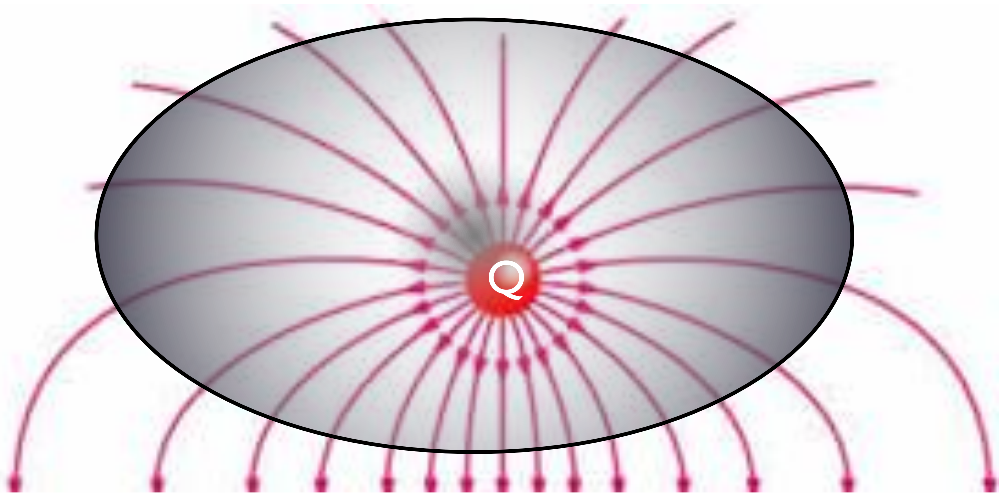
$$\left.\begin{aligned} \frac{dG}{dz} - \frac{dH}{dy} &= \mu\alpha \\ \frac{dH}{dx} - \frac{dF}{dz} &= \mu\beta \\ \frac{dF}{dy} - \frac{dG}{dx} &= \mu\gamma \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (55),$$

with the conditions

$$\frac{1}{4\pi}\left(\frac{d}{dx}\mu\alpha + \frac{d}{dy}\mu\beta + \frac{d}{dz}\mu\gamma\right) = m = 0 \quad \ldots\ldots\ldots\ldots \quad (56),$$

$$\vec{\nabla} \cdot \mu\vec{H} = 0$$

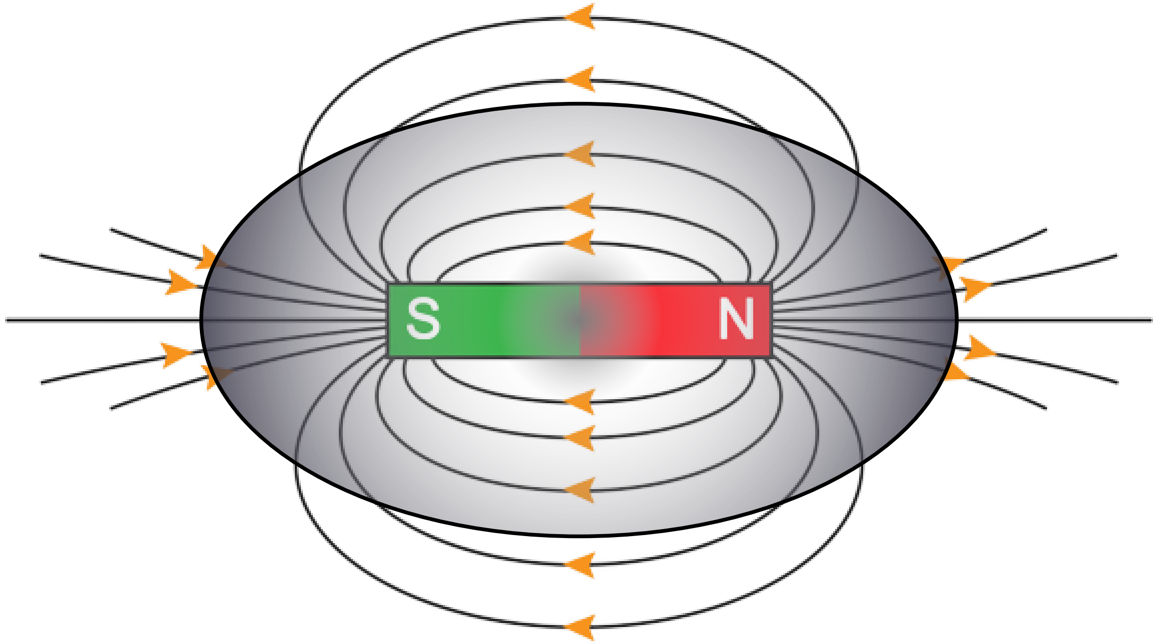
## Gauss's Law




$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon$$

$$\oint \vec{E} \cdot d\vec{s} = Q/\epsilon$$

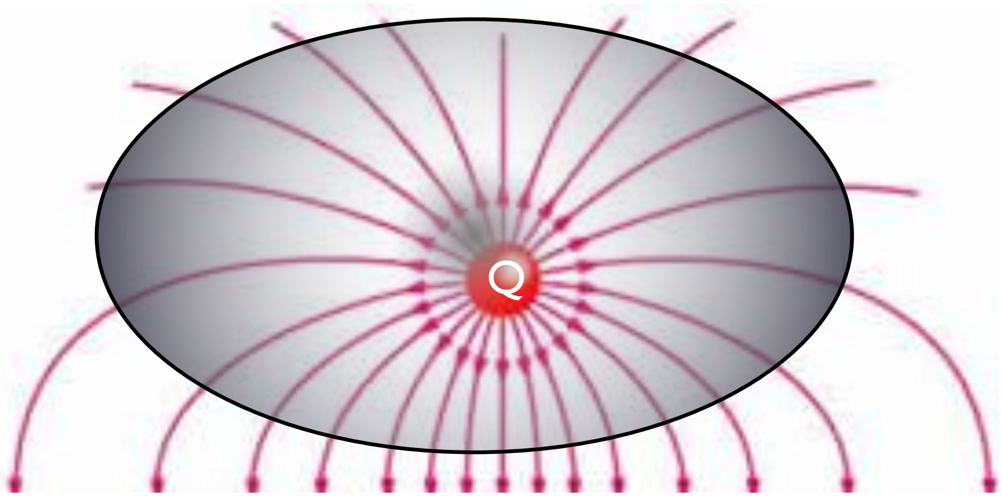
## Gauss's Law for Magnets




$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$

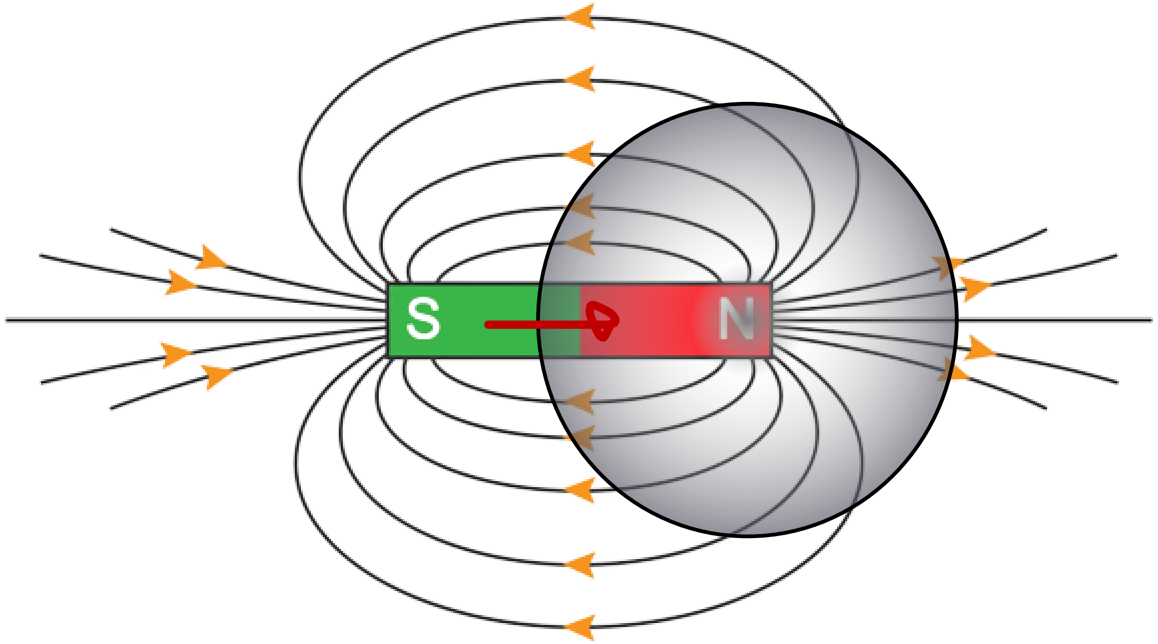
## Gauss's Law




$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{s} = Q/\epsilon$$

## Gauss's Law for Magnets




$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{s} = 0$$

# Part 4

## Faraday, Maxwell & Light

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

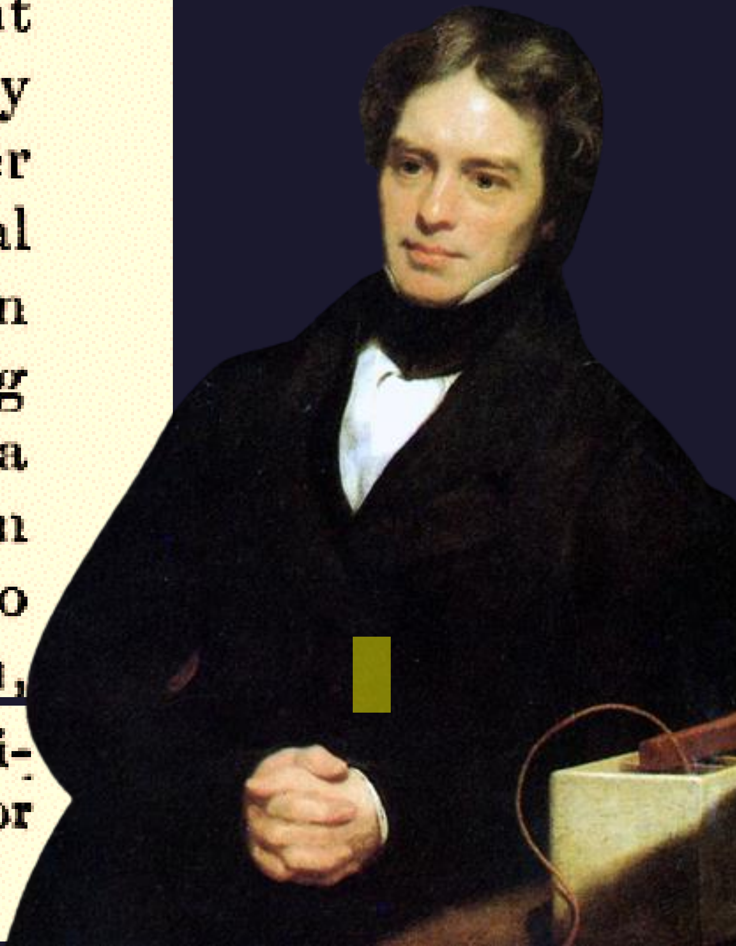
$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{d\vec{E}}{dt}$$

§ 1. *On the Induction of Electric Currents.* § 2. *On the Evolution of Electricity from Magnetism.* § 3. *On a new Electrical Condition of Matter.* § 4. *On* Arago's *Magnetic Phenomena.*

[Read November 24, 1831.]

114. The relation which holds between the magnetic pole, the moving wire or metal, and the direction of the current evolved, *i. e. the law* which governs the evolution of electricity by magneto-electric induction, is very simple, although rather difficult to express. If in fig. 24. PN represent a horizontal wire passing by a marked magnetic pole, so that the direction of its motion shall coincide with the curved line proceeding from below upwards; or if its motion parallel to itself be in a line tangential to the curved line, but in the general direction of the arrows; or if it pass the pole in other directions, but so as to cut the magnetic curves * in the same general direction,

* By magnetic curves I mean the lines of magnetic forces, however modified by the juxtaposition of poles, which would be depicted by iron filings; or those to which a very small magnetic needle would form a tangent.

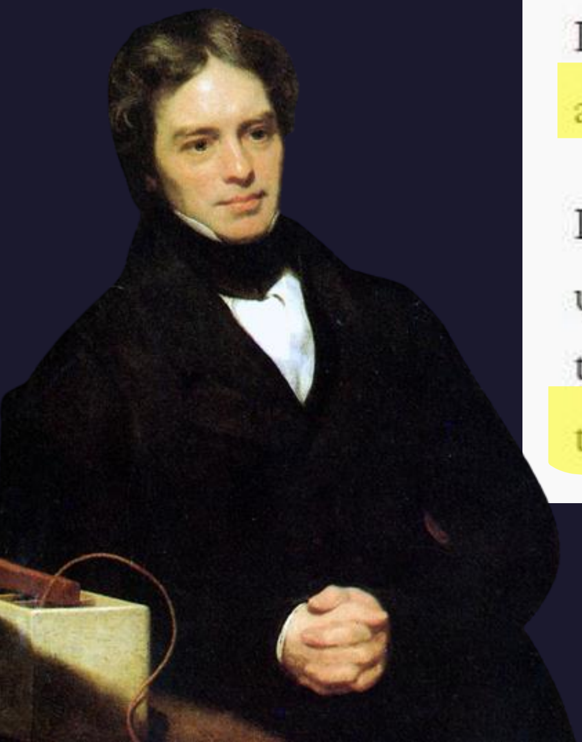


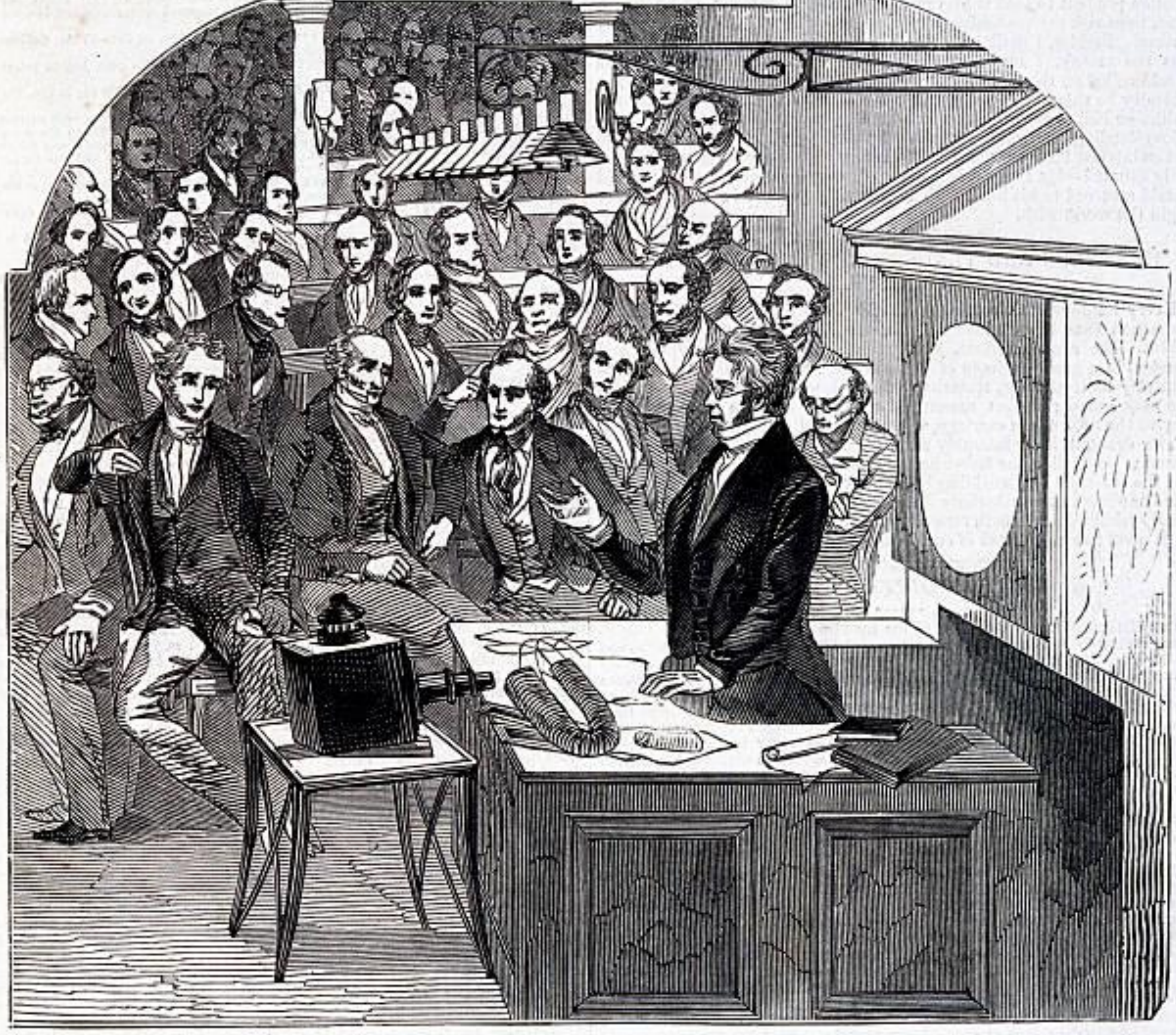
Royal Institution | March 12th 1832

Certain of the results of the investigations which are embodied in the two papers entitled *Experimental researches in Electricity* [2], lately read to the Royal Society, and the views arising therefrom, in connexion with other views and experi-ments, lead me to believe that magnetic action is progressive, and requires time; i.e. that when a magnet acts upon a distant magnet or piece of iron, the influ-encing cause, (which I may for the moment call magnetism,) proceeds gradually from the magnetic bodies, and requires time for its transmission, which will probably be found to be very sensible.

I think also, that I see reason for supposing that electric induction (of tension) is also performed in a similar progressive way.

I am inclined to compare the diffusion of magnetic forces from a magnetic pole, to the vibrations upon the surface of disturbed water, or those of air in the phenomena of sound, i.e., I am inclined to think the vibratory theory will apply to these phenomena, as it does to sound, and most probably to light.





PROFESSOR FARADAY'S LECTURE AT THE ROYAL INSTITUTION.

# May 1846

LIV. *Thoughts on Ray-vibrations.* By MICHAEL FARADAY, Esq., D.C.L., F.R.S., Fullerian Prof., &c. &c.

*To Richard Phillips, Esq.*

*MAY* 1846.

The view which I am so bold as to put forth considers, therefore, radiation as a high species of vibration in the lines of force which are known to connect particles and also masses of matter together. It endeavours to dismiss the æther, but not the vibrations. The kind of vibration which, I believe, can alone account for the wonderful, varied, and beautiful phænomena of polarization, is not the same as that which occurs on the surface of disturbed water, or the waves of sound
ases or liquids, for the vibrations in these cases are direct,
nd from the centre of action, whereas the former are
It seems to me, that the resultant of two or more
force is in an apt condition for that action which may
dered as equivalent to a *lateral* vibration; whereas an
medium, like the æther, does not appear apt, or more
air or water.

# 1862

$$E = \sqrt{\frac{4\pi k}{\mu}}$$



In air or vacuum $\mu = 1$, and therefore

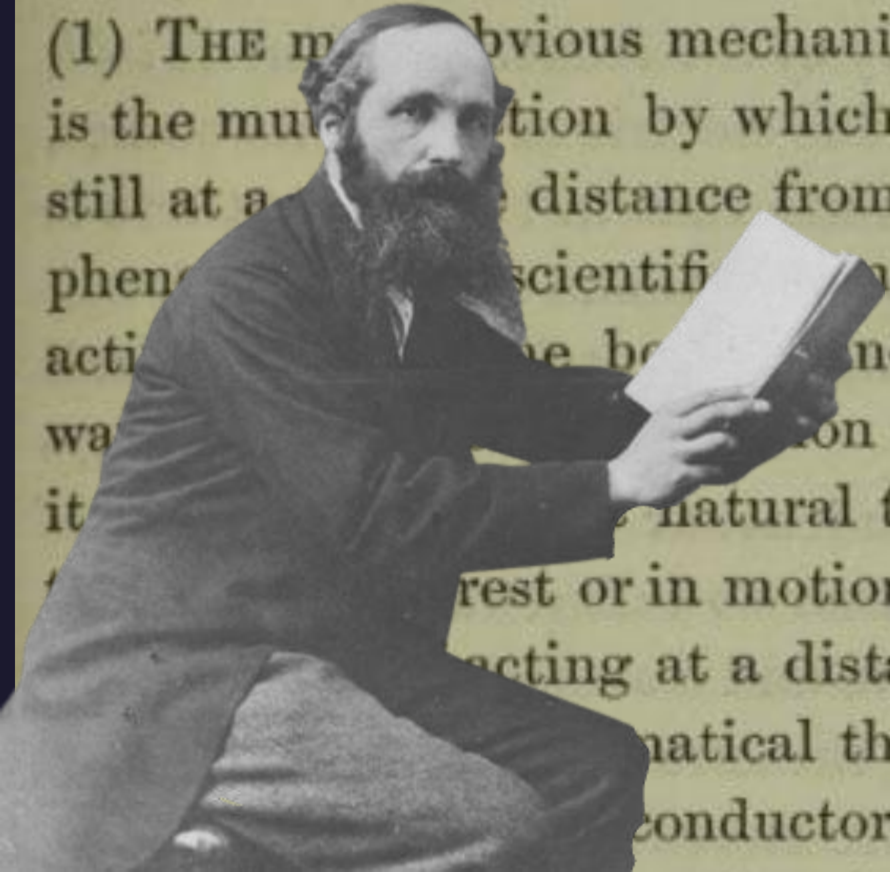
$$\left.\begin{aligned} V &= E \\ &= 310{,}740{,}000{,}000 \text{ millimetres per second} \\ &= 193{,}088 \text{ miles per second} \end{aligned}\right\} \quad (136).$$

The velocity of transverse undulations in our hypothetical medium, calculated from the electro-magnetic experiments of MM. Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.*

## VIII. *A Dynamical Theory of the Electromagnetic Field.* *By* J. Clerk Maxwell, *F.R.S.*

Received October 27,—Read December 8, 1864.

### PART I.—INTRODUCTORY.



(1) The m bvious mechanical phenomenon in electrical and magnetical experiments is the mu tion by which bodies in certain states set each other in motion while still at a distance from each other. The first step, therefore, in reducing these phen scientifi , is to ascertain the magnitude and direction of the force acti e b nd when it is found that this force depends in a certain wa on of the bodies and on their electric or magnetic condition, it natural to explain the facts by assuming the existence of some- rest or in motion in each body, constituting its electric or magnetic state, cting at a distance according to mathematical laws.

natical theories of statical electricity, of magnetism, of the mecha- conductors carrying currents, and of the induction of currents have

1864

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$$

0

# 1864

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$$

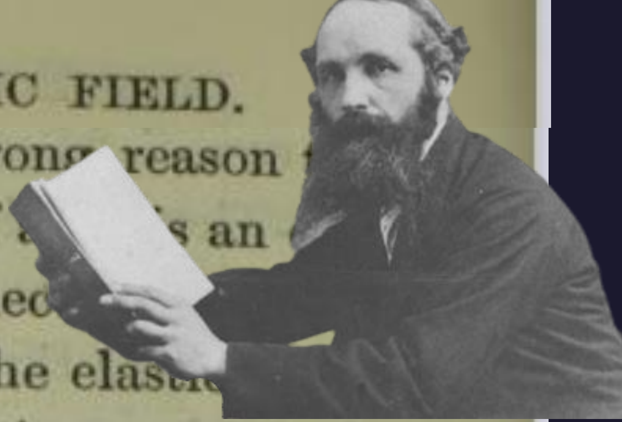
$$c = \sqrt{\frac{4\pi k}{\mu_0}} = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

1864

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{d\vec{E}}{dt}$$

$$c = \sqrt{\frac{4\pi k}{\mu_0}} = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$



This velocity is so nearly that of light, that it seems we have strong reason
clude that light itself (including radiant heat, and other radiations if
magnetic disturbance in the form of waves propagated through the elec
according to electromagnetic laws. If so, the agreement between the elas
medium as calculated from the rapid alternations of luminous vibrations, and as found by the slow processes of electrical experiments, shows how perfect and regular the elastic properties of the medium must be when not encumbered with any matter denser than air. If the same character of the elasticity is retained in dense transparent bodies, it appears that the square of the index of refraction is equal to the product of the specific dielectric capacity and the specific magnetic capacity. Conducting media are shown to absorb such radiations rapidly, and therefore to be generally opaque.

The conception of the propagation of transverse magnetic disturbances to the exclusion of normal ones is distinctly set forth by Professor FARADAY* in his "Thoughts on Ray Vibrations." The electromagnetic theory of light, as proposed by him, is the same in substance as that which I have begun to develope in this paper, except that in 1846 there were no data to calculate the velocity of propagation.

# Part 6

Quaternions, Vectors and 1864

* A communication, substantially the same with that here published, was made by the present writer to the Royal Irish Academy, at the first meeting of that body after the last summer recess, in November 1843.

*On Quaternions; or on a new System of Imaginaries in Algebra*. By Sir* WILLIAM ROWAN HAMILTON, *LL.D., P.R.I.A., F.R.A.S., Hon. M. R. Soc. Ed. and Dub., Hon. or Corr. M. of the Royal or Imperial Academies of St. Petersburgh, Berlin, Turin, and Paris, Member of the American Academy of Arts and Sciences, and of other Scientific Societies at Home and Abroad, Andrews' Prof. of Astronomy in the University of Dublin, and Royal Astronomer of Ireland.*

[*The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science*, 1844–1850.]

1. Let an expression of the form

$$Q = w + ix + jy + kz$$

be called a *quaternion*, when $w$, $x$, $y$, $z$, which we shall call the four *constituents* of the quaternion Q, denote any real quantities, positive or negative or null, but $i$, $j$, $k$ are symbols of three ima... quantities, which we shall call *imaginary units*, and shall suppose to be unconn... relation with each other; in such a manner that if there be another expr...

$$Q' = w' + ix' + jy' + kz',$$

t... ween these two quaternions,

$$Q = Q',$$

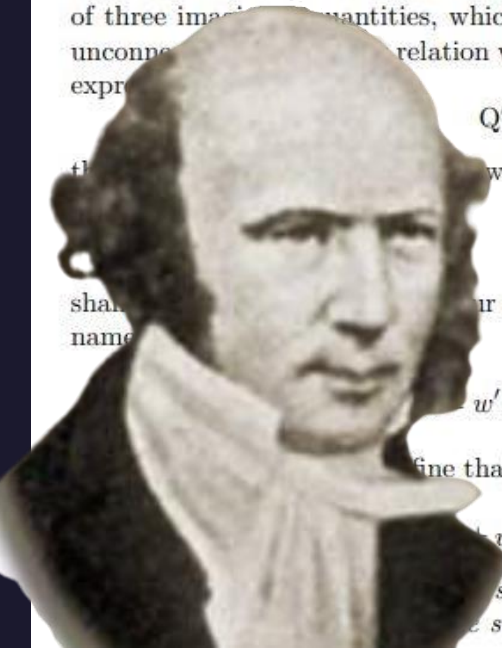
sha... r separate equations between their respective constituents, name...

$$w', \quad x = x', \quad y = y', \quad z = z'.$$

...ne that the *addition* or *subtraction* of quaternions is effected by

$$\pm w' + i(x \pm x') + j(y \pm y') + k(z \pm z');$$

...*sums or differences of the constituents of any two quater-* ...*sum or difference of those two quaternions themselves.* It



Clarendon Press Series

AN ELEMENTARY TREATISE ON

# QUATERNIONS

1869

BY

P. G. TAIT, M. A.

FORMERLY FELLOW OF ST. PETER'S COLLEGE, CAMBRIDG...

PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF...

τετρακτύν,

παγὰν ἀενάου φύσεως ῥιζώματ' ἔχουσαν.

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M.DCC...

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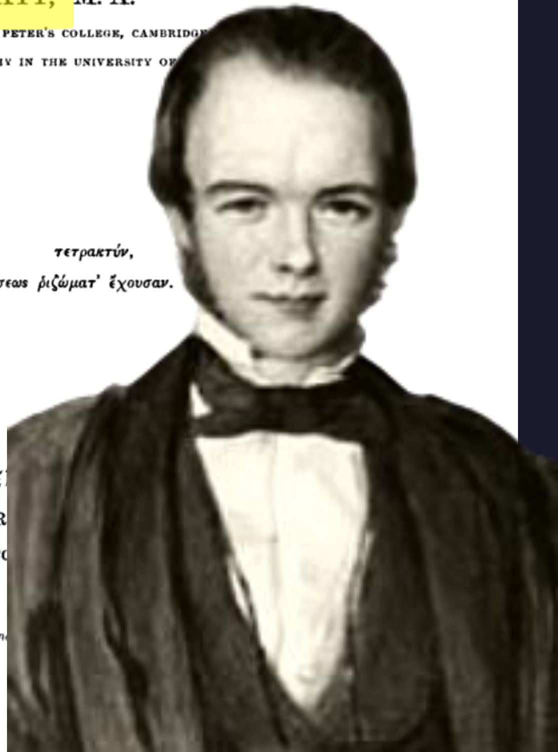


*On Quaternions; or on a new System of Imaginaries in Algebra*. By* Sir WILLIAM ROWAN HAMILTON, *LL.D., P.R.I.A., F.R.A.S., Hon. M. R. Soc. Ed. and Dub., Hon. or Corr. M. of the Royal or Imperial Academies of St. Petersburgh, Berlin, Turin, and Paris, Member of the American Academy of Arts and Sciences, and of other Scientific Societies at Home and Abroad, Andrews' Prof. of Astronomy in the University of Dublin, and Royal Astronomer of Ireland.*

[*The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science*, 1844–1850.]

1. Let an expression of the form

$$Q = w + ix + jy + kz$$

$$\nabla = \frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k$$

Scalar $\nabla A$: $-(\vec{\nabla} \cdot \vec{A})$

Vector $\nabla A$: $(\vec{\nabla} \times \vec{A})$

GLENLAIR, DALBEATTIE,
*Nov. 7, 1870.*

Dear Tait

$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}.$$

What do you call this? Atled?

I want to get a name or names for the result of it on scalar or vector functions of the vector of a point.

Here are some rough hewn names. Will you like a good Divinity shape their ends properly so as to make them stick?

$V\nabla \,.\, \nabla F$ is the twirl of the slope which is necessarily zero

$S\nabla \,.\, \nabla F = \nabla^2 F$ is the convergence of the slope, which is the concentration of $F$.

Also $S\nabla\sigma$ is the convergence of $\sigma$

$V\nabla\sigma$ is the twirl of $\sigma$.

Now, the convergence being a scalar if we operate on it with $\nabla$, we find that it has a slope but no twirl.

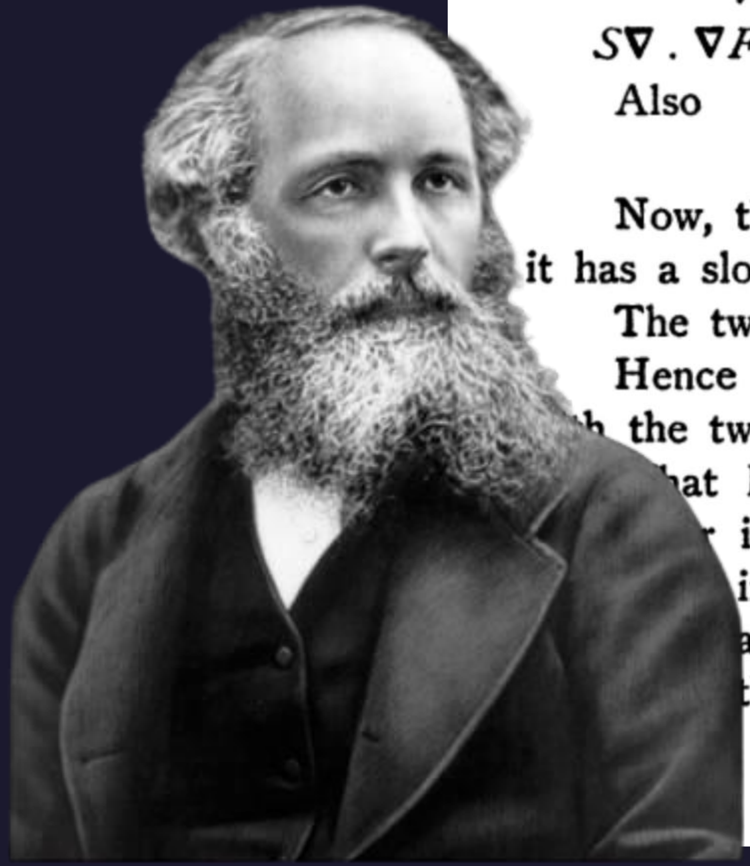
The twirl of $\sigma$ is a vector function which has no convergence but only a twirl.

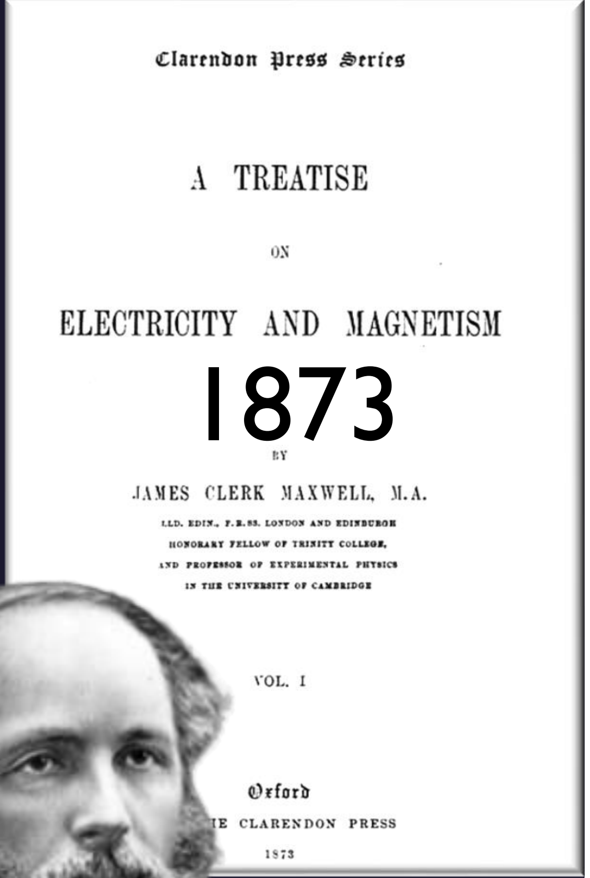
Hence $\nabla^2\sigma$, the concentration of $\sigma$, is the slope of the convergence of $\sigma$ together
h the twirl of the twirl of $\sigma$, the sum of two vectors.

hat I want is to ascertain from you if there are any better names for these
r if these names are inconsistent with anything in Quaternions, for I am
in quaternion idioms and may make solecisms. I want phrases of this
ake statements in electromagnetism and I do not wish to expose either
the contempt of the initiated, or Quaternions to the scorn of the profane.

Yours truly

J. CLERK MAXWELL.




Clarendon Press Series

A TREATISE

ON

ELECTRICITY AND MAGNETISM

1873

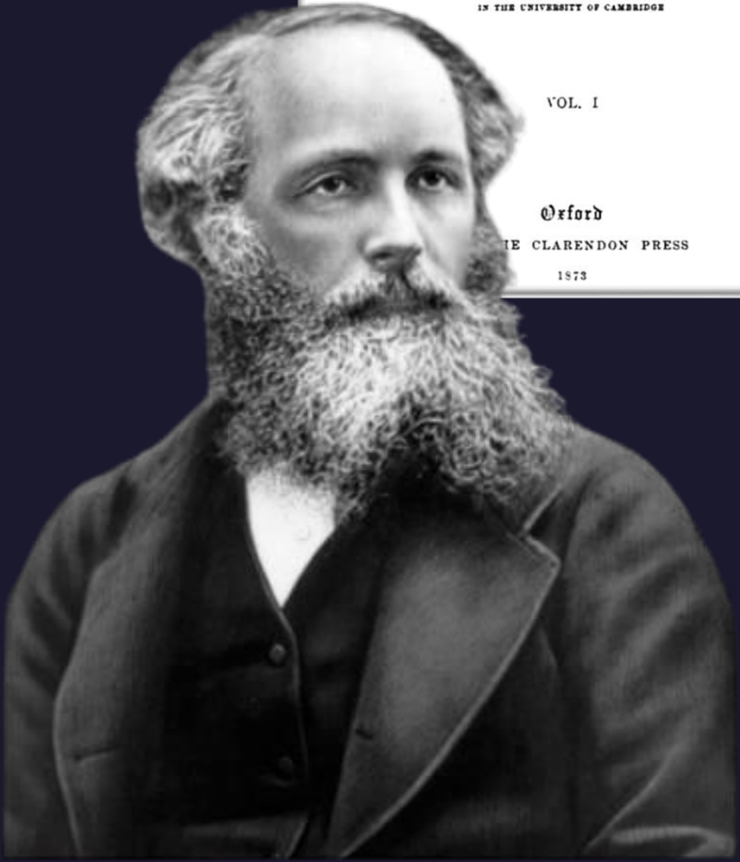
BY

JAMES CLERK MAXWELL, M.A.

LLD. EDIN., F.R.SS. LONDON AND EDINBURGH
HONORARY FELLOW OF TRINITY COLLEGE,
AND PROFESSOR OF EXPERIMENTAL PHYSICS
IN THE UNIVERSITY OF CAMBRIDGE

VOL. I

Oxford
[TH]E CLARENDON PRESS
1873



To understand the meaning of these functions of a vector, let us suppose that $\sigma_0$ is the value of $\sigma$ at a point $P$, and let us examine the value of $\sigma - \sigma_0$ in the neighbourhood of $P$. If we draw a closed surface round $P$, then, if the surface-integral of $\sigma$ over this surface is directed inwards, $S \nabla \sigma$ will be positive, and the vector $\sigma - \sigma_0$ near the point $P$ will be on the whole directed towards $P$, as in the figure (1).

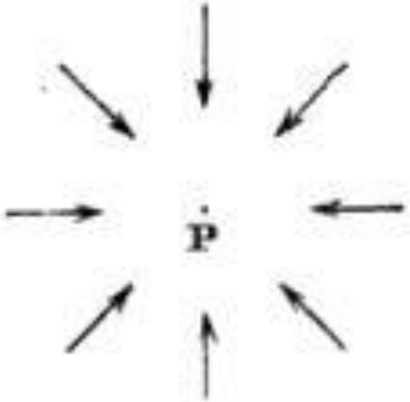


Fig. 1.

I propose therefore to call the scalar part of $\nabla \sigma$ the *convergence* of $\sigma$ at the point $P$.

To interpret the vector part of $\nabla \sigma$, let us suppose ourselves to be looking in the direction of the vector whose components are $\xi, \eta, \zeta$ and let us examine the vector $\sigma - \sigma_0$ near the point $P$. It will appear as in the figure (2), this vector being arranged on the whole tangentially in the direction opposite to the hands of a watch.

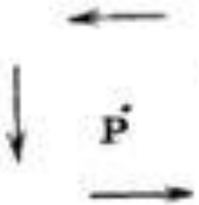


Fig. 2.

I propose (with great diffidence) to call the vector part of $\nabla \sigma$ the *curl*, or the *version* of $\sigma$ at the point $P$.



*I remember my first look at the great treatise of Maxwell's when I was a young man. Up to that time there was not a single comprehensive theory, just a few scraps; I was struggling to understand electricity in the midst of a great obscurity. When I saw on the table in the library the work that had just been published (1873), I browsed through it and I was astonished ! I read the preface and the last chapter, and several bits here and there; I saw that it was great, greater and greatest, with prodigious possibilities in its power.*

*I was determined to master the book and set to work. I was very ignorant. I had no knowledge of mathematical analysis (having learned only school algebra and trigonometry, which I had largely forgotten), and thus my work was laid out for me. It took me several years before I could understand as much as I possibly could. Then I set Maxwell aside and followed my own course. And I progressed much more quickly.*

# ELECTRIC WAVES

BEING

RESEARCHES ON THE PROPAGATION OF ELECTRIC ACTION WITH FINITE VELOCITY THROUGH SPACE

BY

DR. HEINRICH HERTZ

PROFESSOR OF PHYSICS IN THE UNIVERSITY OF BONN

AUTH... SLATION

DIRECTOR... ...NTY COUNCIL
LATELY PROF... ...S, ABERYSTWYTH

WITH ... D., D.C.L.
PRESID... ...ILOSOPHY
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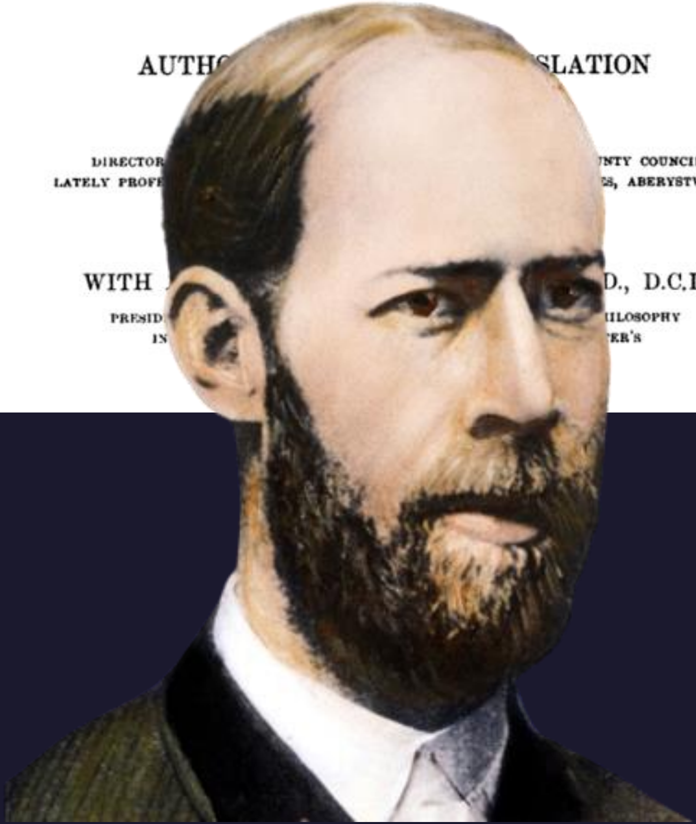
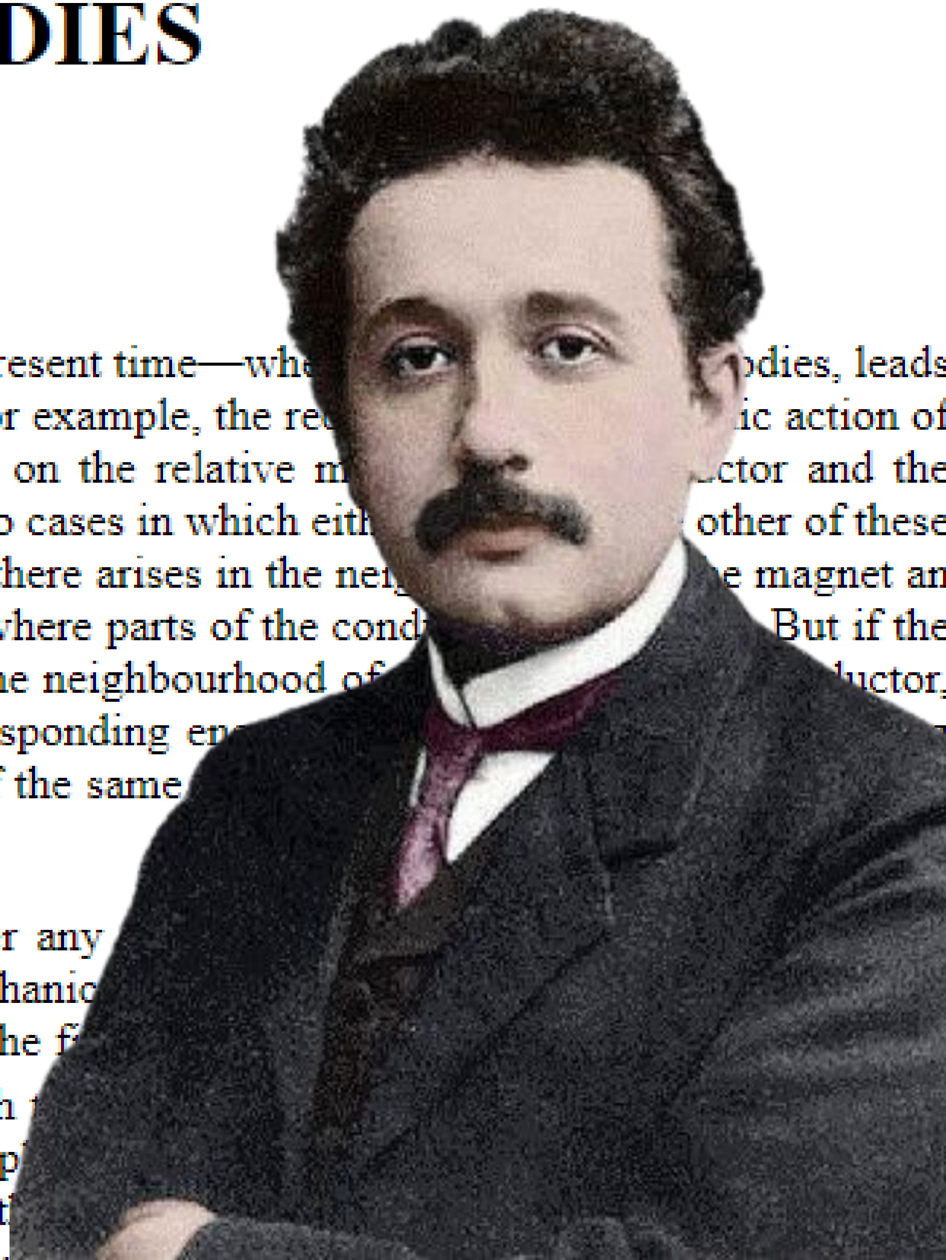




ness of particular theories. Nevertheless, there is an obvious connection between the experiments and the theory in connection with which they were really undertaken. Since the year 1861 science has been in possession of a theory which Maxwell constructed upon Faraday's views, and which we therefore call the Faraday-Maxwell theory. This theory affirms the possibility of the class of phenomena here discovered just as positively as the remaining electrical theories are compelled to deny it. From the outset Maxwell's theory excelled all others and did not rest upon the evidence of decisive experiments. In this connection we can best characterise the object and the result of our experiments by saying: The object of these experiments was to test the fundamental hypotheses of the Faraday-Maxwell theory, and the result of the experiments is to confirm the fundamental hypotheses of the theory.

# ON THE ELECTRODYNAMICS OF MOVING BODIES

**By A. Einstein**
**June 30, 1905**



It is known that Maxwell's electrodynamics—as usually understood at the present time—whe … odies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the re … ic action of a magnet and a conductor. The observable phenomenon here depends only on the relative m … ctor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which eith … other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the nei … e magnet an electric field with a certain definite energy, producing a current at the places where parts of the cond … But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of … luctor, however, we find an electromotive force, to which in itself there is no corresponding en … equality of relative motion in the two cases discussed—to electric currents of the same … the electric forces in the former case.

Examples of this sort, together with the unsuccessful attempts to discover any … medium," suggest that the phenomena of electrodynamics as well as of mechanic … idea of absolute rest. They suggest rather that, as has already been shown to the f … electrodynamics and optics will be valid for all frames of reference for which … raise this conjecture (the purport of which will hereafter be called the "Princip … also introduce another postulate, which is only apparently irreconcilable with t …

# Want to Know More?

- YouTube: "Kathy Loves Physics and History"
- www.KathyLovesPhysics.com
- "The Lighting Tamers" Kathy Joseph